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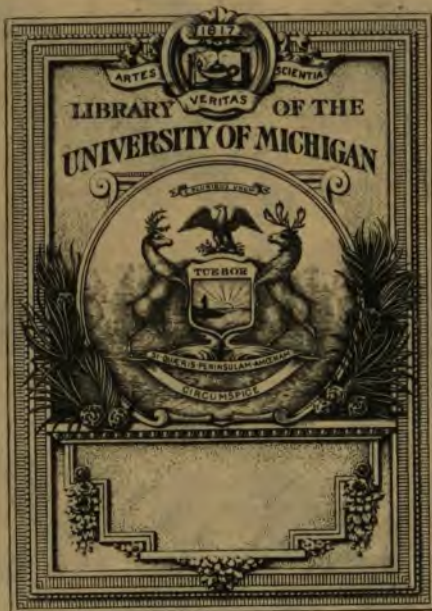
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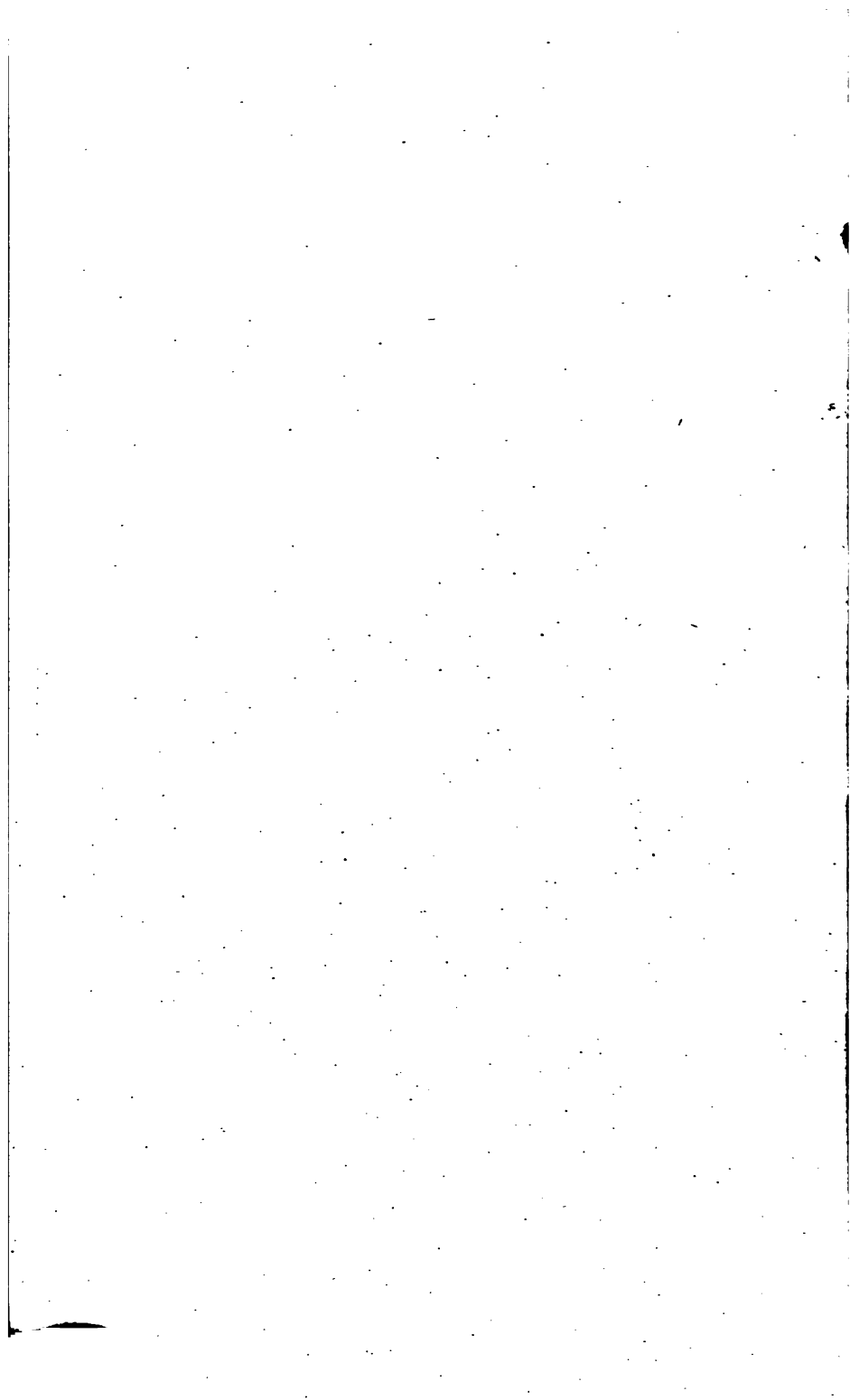
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A TEXT-BOOK FOR SCHOOLS, AND ARTISANS' CLASSES; AND FOR
SELF-INSTRUCTION.

manual
By S. EDWARD WARREN, C. E.

FORMER PROFESSOR IN THE RENSSELAER POLYTECHNIC INSTITUTE, AND MASS. INST. OF
TECHNOLOGY; AND BOSTON NORMAL ART SCHOOL; AND AUTHOR OF A COMPLETE
SERIES OF TEXT-BOOKS ON DESCRIPTIVE GEOMETRY AND INSTRUMENTAL DRAWING.

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PREFACE.

GEOMETRICAL DRAWING is the proper language of Geometrical Design; while the qualifications conferred by common schools greatly assist nearly every one in conceiving designs, at least for simple dwellings, various minor articles, home improvements, and common conveniences, etc., if not for important inventions.

In the former cases it may be a great convenience, and in the latter one almost a necessity, to be able to express one's designs by intelligible drawings. The power, however, both to make and to understand these drawings—that is, to *write and to read the graphic language*, must be acquired by an orderly study of Geometrical Drawing, beginning with the rudiments. Such study would, therefore, seem fitted to be everywhere, as now in some places, a useful element in common education.

The present work, commenced several years since, has grown from humble beginnings, in teaching a very elementary course, to include topics so numerous, and calling for so much care in arrangement and detail, as to suggest that the preparation of a worthy elementary text-book is by no means a holiday task, or one to be performed any less carefully and studiously than the making of a treatise on the higher portions of the same department of study. In fact, the details of right beginnings, though so important, acquire such a commonplace familiarity to a teacher, as to be overlooked with harmful facility when composing a text-book, unless promptly noted as they are suggested in the course of previous experience.

The perfecting of a life-boat, or a steam-governor, may have cost as much thought as the general design of a vessel or a steam-engine; and it is much the same in comparing the composition of a rudimentary and an advanced text-book.

But to pass briefly to a more particular notice of the contents of this manual.

Its four divisions comprise:—*First*, a full description of the familiar, and the more rarely used drawing instruments; *Second*, a complete study of the various operations of the draftsman, considered abstractly, *i.e.*, apart from their application to the drawing of particular structures. Experience as shown the value of this mode of treatment, inasmuch as, according to it, the mind is wholly devoted to one thing at a time, and hence relieved from perplexity; and inasmuch, also, as no time, materials,

or patience are lost by spoiling in *execution*, drawings which have been accurately *constructed* at the cost of some hours or days of labor; *Third*, comes a short collection of practical constructions involving two dimensions, *Fourth*, to complete the draftsman's ability to finish with neatness, correctness, and taste, the previously outlined drawing, some remarks on the principles of taste which are to be observed in making geometrical drawings have been added, which will, it is hoped, contribute materially to the object proposed.

Some topics in the last division, as that concerning colors, may be considered as being treated with unnecessary fulness. But it can never be disadvantageous to an accomplished draftsman to possess cultivated taste, or otherwise than probably very useful to have his knowledge of principles of taste in advance of his foreseen application of them. In respect to the *third* division, especially, I remark that it is not necessary to consistency in a course of drawing of two dimensions, that nothing but surface objects, as figures of two dimensions, should be drawn, but only that no more than two dimensions of the object represented should be considered. Thus, just so far as solids are combined, as in pavements, fronts of brick walls, or arch fronts, with reference to forming a prescribed *superficial pattern* or arrangement, they form proper subjects for drawings involving but two dimensions. The combination, likewise, of spaces or volumes, as rooms in a dwelling, with reference solely to forming a certain ground plan, gives occasion to many entertaining problems of two dimensions, which will probably be found on trial not to be useless in the incidental culture of refined taste in matters concerning domestic and social life.

Problems, like those of compound curves, Div. III. Chap. IV., present themselves under a threefold aspect. First, they may be analyzed and resolved into their abstract geometrical problems. Second, they appear each as one concrete problem, thus: Required to form a compound curve composed of circular arcs and tangent at given points to two parallels. But this concrete problem appears in two forms, its *general* or *generic form*, which is the one just stated; and in its *specific, industrial, or technical form*, as when certain proportions in the required figure are prescribed in denominate numbers, as used in the architectural arts.

These problems, when treated abstractly by the separate solutions of their components, properly appear in a course of simple geometrical problems. When in their ultimate or industrial form, they should be given in works treating primarily of the industrial arts in which they are applied. But in their *general* concrete form, as seen in this volume, they may, I think, be advantageously placed in a course of general geometrical drawing of two dimensions; hence I have included some of them in this form in the present volume.

In respect to the use of this work, it may be suggested that the teacher will find a great saving of time and labor in so conducting the graphic exercises in *construction* as to keep all of the class at work on the same thing at the same time—he explaining at the blackboard and dictating

the successive steps of the student's progress, so that, moreover, uniformity will be secured, and many mistakes will be prevented.

It is true that the advantages of this method have, apparently, a compensating evil, which consists in its partial discouragement of self-reliance, resulting in a certain degree of fancied helplessness on the part of the student. But this evil, it is believed, will not be appreciable when students are properly instructed, and encouraged duly to exercise their own powers, and when they aim so to do.

In the instruction of a class with reference to all matters of mere *execution*, after giving to its members, collectively, such general instructions as can thus be profitably conveyed, the time must, to secure the best results, be chiefly spent in giving immediate personal instruction to each one separately.

PREFACE TO THE REVISED EDITION.

SINCE writing the foregoing preface, fourteen years ago, great changes have taken place in the field of education. The number of Technical Scientific Schools, both privately supported and nationally endowed, has increased many fold. In the meantime, Secondary Education, while not abandoning or diminishing its former distinctive work of "preparation for business or college," has been slowly beginning to shape itself to the other and complementary side of its total proper work, viz., *preparation for industrial life, and for higher scientific study*. Accordingly, quite a number of schools of high grade are now found, which are either wholly or partly devoted to this form of preparatory work.

Nothing seems more obvious, in this connection, than that *similar grades of all preparatory studies* should be taught in all preparatory scientific schools, or departments; and that the higher Technical Schools should thus be enabled to require for admission the elements of *all* the subjects pursued in them, instead of being compelled, as they generally have been hitherto, to do some portions of preparatory work themselves.

Agreeably with these views, the elementary series on *instrumental, or mathematical industrial drawing*, of which this volume is one, is intended for only provisional use in the Higher Technical Schools, until such time as its subjects can be required for admission therein; while its normal intended use is in connection with *preparatory scientific education*, whether for the many mechanical industries, or for subsequent higher scientific study.

The series is also intended for Artisans' Schools; Normal Schools, in training sections of their pupils for teaching its subjects; and for self-instruction.

In thoroughly revising this volume with reference to the several uses just indicated, the following improvements have been made:

1. Considerable new and useful information has, in a very brief space, been wrought into the Division on Instruments and Materials, embracing additional figures, and one new plate, together with an improved rewriting of portions of the former text.

2. Numerous new or improved paragraphs, with *additional practical exercises*, have been scattered through the three remaining divisions.

3. As an undoubted improvement, rendering the work more complete, both in itself and as a preparation for the "PROJECTION DRAWING," with which it is most closely connected, a selection of fifty of the most commonly useful *plane geometrical problems*, from my separate volume on that subject, has been added to Division III.

4. Two plates have been added to Division III., to supplement the woodcuts, and to serve better as models of execution.

Liberal as has been the favor accorded to this volume, in the comparatively limited field hitherto found for it, the progress of industrial and technical education will, it is hoped, secure a much wider field in the future, in which it may, as now improved, be found worthy of corresponding favor.

NEWTON, Mass., October, 1878.

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NOTE.

Pupils' and Students' Outfits.

ONE possible hindrance to the greatly needed more general introduction of elementary instrumental drawing into preparatory schools, being the supposed cost of instruments, it becomes highly important to state here that dealers, who may be found in all the more important industrial centres, are giving increased attention to the furnishing of cheap yet sufficient school sets.

While it is true that the fine work of professional draftsmen requires the best instruments, and that these are expensive, the requirements of pupils' work can be cheaply met. "*School sets*" can be had, including a scale, in most cases, at prices ranging from \$0.75 to \$5.00; the *drawing-board*, *T-square* and *triangles* can be made, if need be, by any good mechanic; and *paper*, bought in quantities for a school, is quite cheap.

Once more:—As this volume, and the preceding one on Geometrical Free-Hand Drawing, may be used together with especial advantage in the manual training of the draftsman and geometrical industrial designer, many of their examples can be used interchangeably: those of this volume, especially from *Division III.*, as additional examples in the free-hand drawing of geometrical objects; and some of those of the other volume as further exercises for instrumental drawing of surface objects.

DRAFTING INSTRUMENTS AND OPERATIONS.

DIVISION I.

INSTRUMENTS AND MATERIALS.

CHAPTER I.

DESCRIPTION OF INSTRUMENTS AND MATERIALS.

Preliminary Remarks.

1. We shall first describe the instruments and materials used in geometrical drawing, in their natural order, as follows:

A. Those connected with the *reception and support* of the drawing; paper, drawing-boards, etc.

B. Those used in *construction*; rules, compasses, scales, etc.

C. Those used in *execution*; pens, ink, brushes, etc.

D. *Accessory articles.*

A. *Paper, Drawing-boards, etc., and their management.*

2. Drawing-paper and cloth is of various forms and kinds, as may be most readily seen by a tabular view, as follows:

The prices are only given as a general guide to those who are unfamiliar with the subject. They will, of course, vary more or less with the conditions of trade and currency.

I. In Sheets.

1°. Whatman's drawing-paper, the standard, with these varieties: cold pressed or rough; hot pressed or smooth; extra rough; extra thick; *selected, or second quality*, of which the latter is considerably cheaper and sufficiently perfect for many purposes.

Name.	Size in Inches.	Prices per Quire.
		\$ \$
Cap	13 × 17	0.70 to 0.80
Demy	15 × 20	0.85 — 1.00
Medium	17 × 22	1.25 — 1.50
Royal	19 × 24	1.60 — 2.00

Name.	Size in Inches.	Prices per Quire.	
		\$	\$
Super-royal.....	19 × 27	2.00	to 2.50
Elephant.....	23 × 28	3.00	— 3.50
Imperial.....	22 × 30	3.00	— 3.50
Columbia.....	23 × 34	4.50	— 5.25
Atlas.....	26 × 34	4.50	— 5.25
Double elephant.....	27 × 40	5.75	— 6.50
Antiquarian.....	31 × 53	18.00	— 30.00

2°. Antique or egg-shell paper, from *demy* to *imperial* sizes.

3°. Machine-made paper. This is of tough surface, standing the rubber well, and is well suited to ordinary school uses, at about half the price of the corresponding sizes (cap, to double elephant) of Whatman's.

4°. English, French, and German tinted papers, for fancy purposes, crayon and charcoal drawing, etc.

5°. Bristol board; of various thicknesses, and white or tinted; composed of sheets stuck and pressed together, and varying in size from cap, 12 × 15 inches, to imperial, 20 × 28 inches.

6°. Tracing-paper, a semi-transparent tissue-paper, for copying purposes. Some kinds, very tough, as French vegetable paper, are durable. Sizes from cap, 13 × 17, to 30 × 40 inches, and price from \$1.20 to \$4.50 per quire.

7°. Blank book, or heavy writing-paper; of cap to imperial sizes, may be used for line drawings.

8°. Whatman's, mounted on cotton; royal to antiquarian sizes.

9°. Transfer-paper, having a smeared surface, capable of making an impression under pressure, and thence used to transfer drawings to new sheets.

II. *In Rolls.*

1°. Detail paper, tough surface, buff color, from 36 to 54 inches wide, 10 to 25 cents per yard.

2°. White or cartoon paper; thin; medium, thick; of various widths and qualities, and from 25 to 70 cents per yard.

3°. Egg-shell paper in rolls, with varieties as above.

4°. Mounted on cloth, different widths from 36 inches to 58 inches, and qualities, \$1.00 to \$2.00 per yard.

5°. Tracing-cloth, more durable than tracing-paper, 18, 30, 36, 42 inches wide.

6°. Tracing-paper. French and German, 40 to 54 inches wide.

3. *Determination of choice.* Various comparatively trifling circumstances, such as the size of desks provided, or a desire to make school practice closely imitate office practice, in the scales adopted (though this is altogether unnecessary), may determine the sizes of paper chosen. But it may usefully guide beginners to add the following :

4. *The sheets from medium to super-royal*, marked off in four plates to a sheet, are highly serviceable in elementary practice ; where it is desirable to have each plate so small that the spoiling of it shall not be a serious hindrance. Where a stouter paper is preferred, double-elephant paper, cut crosswise into two sheets, each 20×27 inches, may replace either of the foregoing.

5. *Super-royal, or Imperial*, in two plates, are, either of them, well adapted for machine drawing, or stone-cutting plates ; and for wall exhibitions of school work.

6. *Elephant, Atlas, or Columbia*, are well suited to bridge or other structure drawings, on usual scales ; while the roll papers are adapted to lecture drawings, or other extra large constructions ; or for long maps—as railway and canal maps.

7. *Detail or office paper*, a buff manila paper, of smooth firm surface, is useful and sufficient, when, for the sake of securing quantity and variety of practice, rather than finished execution, a part of the required drawings are left in pencil, or in inked lines, with little or no shading or coloring.

8. *Tracing-cloth*, being durable, is convenient for portable drawings, as builders' plans ; and those which are to be folded up with manuscript ; also for making permanent copies of standard drawings made on paper.

9. *Paper mounted on cloth* is very durable, and adapted to drawings for repeated lectures, office preservation, and for any other purposes in which permanence is desirable.

10. *The flat writing-paper*, obtainable at book and stationery stores, and paper warehouses, is of various weights ; and, as to texture, is "*wove*," having a uniform surface, or "*laid*," having a wavy or water-lined appearance.

11. *Drawing-boards*, with respect to material, are usually of pine or mahogany.

The size best adapted for all the uses described in this course, and for instruction generally in geometrical drawing, is 20×28 inches. A mahogany board of this size need not be over half an inch thick, and a pine board $\frac{3}{4}$ of an inch thick. The material should be thoroughly seasoned, and free from pitchy spots, and the

board should be bound at both ends with cleats of *hard wood*, tongued and grooved into the body of the board to prevent warping.

12. *For drawings on small plates*, particularly when it is desired to carry the drawing to and from school frequently, a small thin board, about 12 × 15 inches in size, is very convenient. These, and indeed, all drawing-boards, can be made of very enduring form, by gluing together many narrow pieces.

13. Drawing-boards are sometimes made to rest in a rectangular frame, just as the glass does in a picture-frame, and are then secured with buttons, or cross bars; but this arrangement is not as good as the solid pine board above mentioned, since only one size of paper can be advantageously used in it.

14. Little pins, with broad brass or German-silver heads, and called *Thumb-tacks*, are used for securing paper loosely to the drawing-board. See Pl. I., Fig. 10.

15. *For firmly securing the paper to the drawing-board*, a process which will be fully explained further on, nothing is more convenient than the "mucilage," now everywhere cheaply sold in bottles of convenient form, together with a suitable stiff flat brush for its application. This substance has quite superseded the dissolved gum-arabic, prepared by each one for himself, which was used in former years.

§ B.—Of the Instruments used in Construction.

16. *Compasses, Rulers, and Scales* summarily embrace the instruments here to be described.

Compasses, or dividers, Pl. I., Fig. 1, are instruments with two movable legs, turning on an axis, and in one plane. They are for the purpose of laying down related pairs of points upon the paper, and for drawing circles, and curves compounded of circular arcs.

Rulers are instruments for drawing lines, whether straight or curved, on the principle of following a continuous guide.

Scales are for the purpose of determining measures, whether linear or angular.

17. *In exact drawing*, as in the construction of a geometrical figure, many or most of the lines are drawn with the ruler, through points *previously located* by the use of the *compasses*. When, in addition to a *given law of construction*, as in the case supposed, the drawing is made for some *industrial purpose*, and there-

fore from *given measurements*, definite distances are taken from *scales*.

We shall, therefore, describe the above instruments in what appears to be their natural order, *compasses, rulers, scales*.

§ a. *Compasses, or Dividers.*

18. *General remarks. Names.* So far as compasses need to be distinguished from dividers, the latter term seems to be properly confined to compasses with unremovable steel points only, as if in Pl. I., Fig. 1, the clamp-screws, *hh*, were to be omitted, and the points *a* and *b* made solid with the stock *cc*.

Kinds and Qualities. Several grades of metallic drawing, or mathematical instruments are made. Also those of English, French, German, and Swiss manufacture differ from each other in various particulars of design and style.

19. *Swiss instruments* are of two qualities, of which the best are the finest instruments that are made, while the others are good enough for all ordinary purposes.

20. *French and German instruments*, especially the former, are very cheap, and sufficient for apprentices and schools, as well as for the more common drafting purposes.

21. *English instruments* have quite a distinct style of their own, but are chiefly distinguished by what is called the *sector joint* in the dividers and compasses, instead of the *long joint*, commonly seen. The latter joint extends down from the head into the body of the instrument, as indicated by the line *mn*, Pl. I., Fig. 1.

No such junction appears in the sector joint, its working surfaces being entirely confined to the head of the instrument. These surfaces, when fitted, are coated by the maker with hot beeswax, which lasts for an indefinite time, and gives a perfect deadness of motion, or freedom from ready displacement, without the necessity of screwing the joint up very tightly with the key *gf*.

These joints, it is claimed, open and shut at every degree of opening, with more perfect uniformity of effort, than is the case with the usual long-joint instrument.*

22. *American instruments*, of the Swiss quality, and known as

* For many details too numerous as well as unnecessary to place in a text-book, though interesting to experts, see Stanley's mathematical drawing instruments.

Altener's, are distinguished by an adjustable *pivot joint*, and are well worthy of attention.

23. *Detailed description.* The various kinds of *compasses* will now be described.

(a.) *The plain dividers* have immovable legs of steel, fastened to a brass or German silver stock. The best kinds have steel joints, capable of being loosened and tightened. These, and all kinds of jointed compasses, should open and close with a uniform resistance at all parts of the motion, and without a jolt due to the imperfect fit of the joint-pin in its bearings, and the points should be sharp, of the same length, and close together when the instrument is shut. These dividers are used to lay off given distances on a line, and to divide a line into equal parts.

(b.) *The hair-spring dividers* have, on the inner side of the stock of one leg, a spring, which is fixed at its upper extremity in a groove, and to it one of the steel points is attached. A screw passes through both the stock and the spring, by which the spring is controlled, so as to move the attached steel point through a very minute space. The use of this instrument is, to take off a distance from a finely graduated scale more readily than by the plain dividers, which are operated by the unassisted hand.

(c.) *The large dividers with movable points*, and their furniture, needle-point, pencil-holder, extension bar, and pen-point. Pl. I., Fig. (1-4).

There are two modes of fastening the movable legs, viz., by clamp-screws and by a "bayonet" insertion, *no*, Pl. II., Fig. 8.

Of these modes, the former is preferable, though less quickly accomplished, since it does not become ineffectual through looseness of parts after long use; though the bayonet, or slip joint, is preferred by some, and if coated with wax as above described (21), may be permanent.

For drawing large circles, an extension leg, not shown, is provided, which is inserted between the pen or pencil point and the stock.

Among the furniture is sometimes found a *dotting-pen*, which consists of a small wheel, armed with points, and placed between the points of two blades, between which, and over the wheel, the ink is placed, as in the drawing-pen.

Sometimes the form of the large compasses is modified by having the needle-point permanently attached to one leg in place of the ordinary three-sided steel point.

(d.) *The small compasses with movable legs* are but a duplicate

of those just described, only much smaller, and used for drawing small circles, down, say to those of one-eighth of an inch radius. Theoretically, this could be done with the large compasses, but practically, even the resistance of smooth paper will spring their points out of place to an amount which, though very small, is large *in proportion* to the radius of a very small circle. By shortening the compasses, the point is brought nearer to the joint, and the whole is thus made stiffer. Compasses of the various sizes, but especially the smaller ones, are often provided with handles at the head to facilitate their use.

The large and small compasses of different makes, vary from $3\frac{1}{2}$ to 7 inches in length; and in price, indicative of quality, from *French brass instruments on cards*, for young pupils, at \$1.00, embracing one pair of $5\frac{1}{2}$ inch compasses with fixed needle-point and other furniture, and a drawing-pen; to *German instruments*, with the same and a scale, at \$3.00 to \$4.00; and the best *Swiss* at \$10.25 for the same number of pieces, for expert draftsmen. These, however, provide a separate needle-point, but do not include the drawing-pen.

(e) Among other forms of small compasses for special uses are the *Bow compasses*, used only for drawing circles with pen or pencil. There are three forms of these compasses. The first form is like the small dividers just described, only it is somewhat smaller, and has a needle-point permanently attached to one leg, and a pen or pencil-point to the other, one or both legs being jointed, so that the working parts may be held vertically. See Pl. II. Fig. 2, which, however, shows a *movable* needle-point.

The second form, see Pl. II Fig. 3, might be called "box bow-pens," since the part *a* is enclosed on all sides, except that on which the pen is seen. This pen is attached to a spring which is screwed at the upper end to the bottom of the box *a*. The screw, *s*, draws in or releases the pen by its action upon the spring which carries the pen.

The pen is jointed at *n*, so that it can be made vertical, and the outer blade is jointed at *p*, allowing it to be opened for convenience in cleaning.

The third form is like the "spacing dividers," Pl. II Fig. 1, except that it carries a pen or pencil in place of one of the needle-points.

The value of these instruments being greatly diminished by breakage of the usual fixed needle-points, it is a great improvement to have them made with movable needle-points.

(f) The *spacing-dividers* are like the third form of bow-pen, except that both legs terminate in fine points, so that the sole use of the instrument is, to take off small distances with accuracy. See Pl. I. Fig. 1.

As a general principle, simplicity in an instrument is very desirable, hence it is preferable to have the spacing-dividers, pen, and pencil as three separate instruments.

Sometimes, however, as shown in Pl. II. Fig. 4, the bow-pencil compass of this form has a collar or sheath, *s*, at about the middle of the length of one of the legs of the instrument. A separate piece, *p*, is formed, with a pen at one end and a needle-point, *c*, at the other, which slides into this sheath, and is held there snugly by a flat spring within the sheath. Thus the three instruments are combined in one, but they are usually longer, and therefore less stiff than the kinds before described; besides, the pen is liable to "wiggle" in its sheath after being used for some time. Another form of spacing-dividers, which is very stiff and will control longer distances than the form already described, is shown in Pl. I. Fig. 5. It has stout triangular legs, joined at the top by a stiff circular spring, *q*, on which is mounted a handle to facilitate its use. [These instruments, and those of section (e) are here described together for convenience. When used as pens, they strictly belong to the class of instruments used in execution.] All compasses, whose legs are thus connected by screws, should have those screws made to work on a pivot, *e*, Figs. 1 and 5, as otherwise the nut, *s*, will bind and tear out the threads after the legs have been separated for a short distance. As in the case of bow-pens, it is a great improvement to have movable needle-points, as *N*, Pl. II. Fig. 3, attached to the spacing-dividers.

(g) *Beam compasses* of various forms are used for drawing, with great accuracy, very large arcs or circles. There are three forms which will here be described.

The first, see Pl. II. Fig. 6, has the "beam," *B*, made of wood, of a prismatic form, so as to keep the axes of the two points parallel. *CC* are sliding-collars, carrying the vertical working parts. Between the outer plates of each of these collars, is a third plate which is clamped tightly against the beam by the screw, *Z*. *gg* are guides attached to this interior plate, and working through holes in the outer plate, so as to keep the interior plate from dropping out or slipping about. The vertical leg, *h*, is attached to a hollow cylinder, *e*, which, with the leg, may be slowly shifted by turning the screwhead, *s*, which bears against the fixed cylinder, *c*. Thus

the distance between the needle-points can be exactly adjusted to a given radius. The other needle-point is released by loosing the screw, *t*, and may be replaced by a pen or a pencil-holder.

There are slight modifications of this construction in other instruments having wooden beams, but the general arrangement is the same. The second form can be understood from the description of the first, without an additional figure. The beam in this case is cylindrical and of metal, formed in lengths which are made to be screwed together. The sliding-collars which carry the working parts are, therefore, hollow cylinders. In order to keep the axes of the working points parallel, a small, straight groove is formed throughout all the pieces of the beam. When the pieces are screwed together, this groove becomes continuous, and a small projection from the inside of the collars, which slides in this groove, keeps the collar from turning on the beam. The working parts need no further mention, being the same as in the last form.

The third form, also called *tubular compasses*, is jointed like plain dividers. Each leg is a hollow tube, from which the lower part can be drawn out as the tubes of a spy-glass are extended. See Pl. II. Figs. 7 and 8. *J* is the joint, of the usual form. *k* is a portion of one leg not fully represented. *a* is the hollow part of the leg, broken so as to show, as at *b*, that it is hollow. From out of this hollow part slides the cylinder, *d*, which is clamped by a screw, *s*, and carries a box, *e*, which receives the pen, pencil-holder, or needle-point, *h*. *gg* is a groove in which slides a guide, screwed into the box, *s*, of the leg, *a*, at *B*. This prevents the cylinder, *d*, from turning in its sheath.

The peculiar fastening of the piece, *h*, to the box, *e*, is an instance of the bayonet-fastening before mentioned. It consists of a plate, *o*, fixed into the cylindrical part, *n*, and a slit, *p*, in the box, *e*, which receives the plate, *o*. It has not the sure tightness of the common screw-fastening, and is but little, if any, more expeditiously used. By opening the legs till the corners, *rr*, meet, and drawing out the tubes to their full extent, a radius of twenty-six or eight inches can be commanded.

(h) *Proportional compasses*.—Pl. II. Fig. 9, shows one of these compasses in its most complete form. This instrument is principally used for expeditiously copying a given figure, so that the copy shall have any desired ratio to the original. This is accomplished by means of *two pairs of points* on opposite sides of the joint, and by making the *joint movable*.

Description.—LL, ll are the legs, of steel. The body L' L' is of German silver. The whole instrument is in two parts, which meet on the line *pp'*; and, as the instrument is shown in the figure, these turn in a plane perpendicular to the surface of the paper, about the *axis ai* of the screw *y* and movable joint UU'. This joint is in two parts, U and U', both fitted to the same pin *ai*. Each half of the body, L and L', contains an open groove, or slot, extending nearly from L to L'. Projections on the inner sides of U and U', extending to the line *pp'*, freely slide in these grooves, when the instrument is closed as shown in the figure.

R is an adjusting bar, which can be clamped by the screw *e*, to the stud T, which enters U' by means of a small pin in its under side. The screw *m* then operates to adjust the slide precisely to any desired position. Also, when the instrument is open, T can be transferred, to rest by its pin in the hollow stud *r*, borne by the under half of the instrument. The bar, B, then connects the two legs, and by means of the screw *m*, operates as in Figs. 1 and 5, to set the points exactly to a desired opening.

Graduation.—The most complete instruments bear four scales, named Lines, Circles, Polygons, Solids. Of these the scale of lines is shown, its divisions expressed in fractions. The sliding joint carries a line or index cut upon it for each scale.

Operation and uses. Scales of Lines.—The joint being set so that a fine line cut upon it at *i*, and called its index, coincides with the division marked $\frac{1}{4}$ on the scale of lines, the opening of the point at *p'* will then always be *one-fourth* of the opening at *p*, whatever this may be. Thus, having a given figure, to be copied at one-fourth of its original size, we *take up* each line of the *given* figure with the points *p*, and set it down upon the *new* figure with the points *p'*. Conversely, if the given figure is to be copied of four times the given size, we should take up the lines of the given figure with the points *p'*, and set them down on the copy with the points *p*. Like explanations apply to all the other divisions of the scale of lines.

Scale of Circles.—This is used for dividing circles into equal parts, in constructing inscribed polygons or for any other purpose. At $\frac{1}{4}$ on the scale of circles, the opening at *p'* will be the chord of *one-fourth* of the circumference of the circle whose radius is the opening at *p*, whatever this may be.

Scale of Polygons, otherwise called that of Plans, or Areas.—(From the last paragraph, the scale of circles might also be not inappropriately called the scale of polygons.) Retaining the same

ratio, $\frac{1}{4}$, for easier comparison, if the joint be set at $\frac{1}{4}$ on the scale of areas, and if the sides of a given figure be taken up with the points p , and set down with the points p' , the *area* of the copy will be *one-fourth* of that of the original.

Scale of Solids.—Setting the joint by its index, at $\frac{1}{4}$ on this scale, and taking up the edge of a given solid with the points at p , the opening of the points at p' will measure the corresponding edge of a similar solid whose volume will be one-fourth of that of the given body.

Square and cube roots.—Knowing that similar *areas* are to each other as the *squares* of their like (homologous) sides; and that similar *solids* are to each other as the *cubes* of their corresponding (homologous) edges, we find that, conversely, the like sides of similar figures are to each other as the *square roots* of their areas; and the like edges of similar solids are to each other as the *cube roots* of those solids. Hence, when the joint is set at $\frac{1}{4}$, for example, on the scale of *areas*, the areas of the similar figures constructed with the points p and p' will be, respectively, as 5 and 1, and their sides will be as the square roots of 5 and of 1 respectively.

Similar reasoning applies to other divisions, and to solids.

The value of this instrument is lessened, but not destroyed, if any of its points are broken, for after setting the joint approximately to a required ratio by the scale, the ratio of the distances between its points can be found by trial, by the aid of the screw, *m*. Still, great care should be taken not to break the points; and having obtained and laid off, *once*, a required distance from them, that distance should be taken up, if it is to be laid down several times, in a pair of plain compasses, and by them laid off upon the drawing, so as to save the points of the proportional compasses from avoidable wear.

(i.) *Whole and half compasses*.—This instrument, Fig. *a*, is a special case or partial form of the last one. It has legs of the ordi-

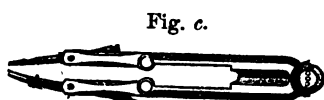
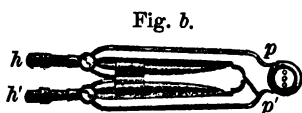
Fig. *a*.



nary form on both sides of a "box-screw" joint of the ordinary construction, the pair of legs on one side being just half as long as the pair on the other side, so that the distance between the points of the longer pair will always be just double the distance between the points of the shorter pair. Hence the instrument can be used to make a drawing of either double or half the size of a given copy.

These instruments, more particularly the proportional compasses, are quite bulky and costly, while their value depends, particularly in the whole and half compasses, on the preservation of the exact ratio between the length of the legs, which ratio is liable to be destroyed by uneven wear or by accidental breakage.

(j.) *Portable compasses*.—These are of three forms: *Pocket dividers, with plain points*. *Pillar Compasses*, Fig. *b*, and *Pocket Compasses*, Fig. *c*.



They are made so as to protect the working parts, and to occupy but little space, so that they can be carried loosely in the pocket; and thus are convenient for use wherever needed.

The plain pocket dividers consist only of a pair of ordinary plain compasses, furnished with a sheath, which encloses both points and is screwed to the stock near to where the steel points are welded in.

The “pillars,” *p* and *p'*, Fig. *b*, are hollow. The handles *h* and *h'* are jointed at their bases, and each, as here shown, carries a steel point between the pillars, and a pen, or pencil-point within the pillars. Thus the handles, when drawn out, afford a small *bow pen*, or bow pencil; while by unfolding, without drawing out, we have a pair of plain dividers, and by inserting one or the other needle-point in the pillars, we have large dividers with pen or pencil point as desired.

These compasses are thus appropriately termed “universal.” Fig. *c* affords only *large* dividers, with needle, pen, or pencil points as desired, by turning the points on a central pin in the slotted legs. The two forms, *b* and *c*, may therefore be distinguished as universal compasses, with *shifting*, and with *swivelling* points.

(k) *Triangular*, or three-legged compasses, have a third leg, hinged to the joint of the other two legs, and capable of turning on the pivot of the principal joint. Hence the three points can be placed in every possible relative position. Hence, again, its use is to copy a map or other drawing, which has been divided up into triangles, by placing one of the points at each vertex of each triangle successively.

Remark. On instruments with separable parts, a sufficient num-

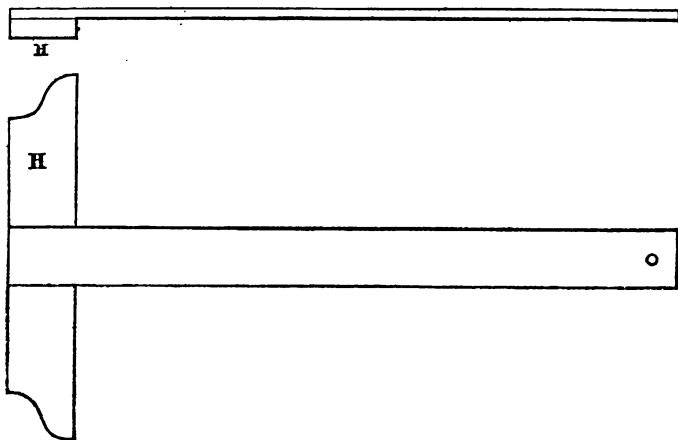
ber of corresponding parts are marked with a black dot, to show how they should be put together.

§ b. Rulers.

24. A drafting office needs various rulers, also called *straight edges*, more than are necessary to the student, and among them some of considerable length. The best are of steel, carefully kept bright, and conveniently held in place, when in use, by heavy weights. Otherwise, they may be made of any fine wood, of permanent form, such as mahogany, maple, or pear-wood. These may be edged with metal; but, otherwise, are thought to be best not varnished, at least on the working edges.

25. A drafting office will be provided with T squares of various sizes, suited to different boards, and to the work in hand. Those suited to the drawing-board here contemplated for students (11), may have a head, H, 10 inches long and $\frac{5}{8}$ of an inch thick, heavy

FIG. 1.



enough to keep in place easily, with a blade as long as the board, 2 inches wide, and $\frac{1}{8}$ of an inch or more in thickness. The blade should be fastened firmly to the upper side of the head, and sunk into it, and should be truly at right angles to it.

This arrangement, it will be readily seen, allows of more freedom in the use of the triangles when they are slid along against the blade of the T square than can be had when the blade is let into

the head, bringing the top of the head above the surface of the drawing-board.

26. When the blade is framed into the head, there is sometimes a second or *movable head* placed above the fixed head, and revolving on a pivot passing through the centre of the fixed head. A screw upon the upper end of this pivot serves to clamp the movable head in a position making any desired angle with the edge of the blade, whence by turning the instrument upside down and by placing the movable head, when in any of its oblique positions, against the end of the board, parallel lines may be drawn in any oblique position across the board.

The objection to this double-headed instrument is, that the movable head can never be clamped so tightly as to prevent a slight rap on the remote end of the blade from disarranging it, while, at the same time, parallel lines can be drawn with great facility in any direction by the use of the two triangles combined, as will shortly be shown.

Again, the head is sometimes made in two pieces, separated by two pieces of wood of the same thickness as the blade, and placed at each side of the blade, so as to leave an open space in the middle, between the two outside pieces of the head, into which open space the blade is fitted, and turns on a bolt passing through it and both pieces of the head. This form has the same uses and objections as the one last described.

The T square may be made of any *fine-grained* hard wood not liable to warp.

The blade is sometimes made of metal, and if a metallic blade could be found which would not rust or be readily tarnished, so as to soil the paper badly, it would be preferable to any other kind.

There are forms of the metallic T square, protracting squares or "Trigonometers," which carry a graduated arc at the head, and a vernier upon the adjacent end of the blade, together with various sliding-scales upon the blade, sometimes, so that they can be used in plotting surveys, by setting the blade at any angle with the head. This brief description will serve to call the attention of the reader to the existence of a T square of this form, which is all that is intended here.

27. A substitute for the T square is the *parallel ruler*. This consists of two rulers connected by parallel metallic arms, which revolve on pivots, so that as the rulers are separated by revolving the arms they will always be parallel; the principle being, that parallel lines have the parallel distances between them equal

28. Another, and in theory, very elegant form of this instrument, consists of a thick heavy ruler, mounted on the common axis of two equal cylindrical rollers, the ruler being cut away for the reception of the rollers. If the rollers are of precisely equal circumference, and do not slip as the instrument is moved, the edge of the ruler will remain parallel to itself, as the instrument is rolled along upon the paper, the axis of the rollers being in the direction of the length of the ruler.

29. *Triangles*, Pl. I., Fig. 6, are of various sizes and proportions, one of the angles being usually a right angle. In respect to *material*, they are made of glass, wood, metal, or hard India-rubber. In respect to *form*, they are solid or open. Metallic and rubber triangles are made open in a single piece. Open wooden triangles are framed together in three pieces. Open and light colored triangles are here recommended, each of them having a right angle, one having also angles of 45° , and the other angles of 60° and 30° .

The sides opposite to the angles of 45° and 60° may each be nine inches long. For fine work, as in the smaller parts of machine drawing, some prefer a pair of triangles of the same proportions as the above, with their corresponding sides about five inches long.

30. The advantages of the open wood triangles are, that they are *more* easily handled and *less* liable to warp. All open triangles also expose the draftsman's work more than solid ones, except glass ones, which are transparent. Rubber or dark wood triangles, being black or dark colored, more readily conceal any ink that may get on them, and thus are more liable to occasion blotting. Metallic triangles soil the paper more than others, though this effect can be prevented by providing them with ivory feet. The framing at the angles of wooden triangles should be carried in quite deep, nearly to the angles of the inner edges in every case. As regards cleanliness, the natural surface of the wood, or a coat of fine polish, is better than common varnish.

31. *Triangles having other acute angles*, than those of 45° , and 30° and 60° , are chiefly useful to architects, in drawing *roof pitches*; and to *engineers*, in drawing *earth slopes*. Their acute angles, for the former purpose, should always bear some simple ratio to a right angle.*

* See the elements of geometric beauty in my *ELEMENTARY FREE-HAND DRAWING. Part III*

Triangles of 90° , $67\frac{1}{2}^\circ$, $22\frac{1}{2}^\circ$ are convenient for some purposes, such as dividing a circle into sixteen equal parts.

32. Lettering triangles are a convenience to those whose facility in lettering is unequal to their other drafting accomplishments. In these, the hypotenuse is made polygonal with four sides, one suited to the slant of K; another to that of N, X, Y; another to that of A, M, V; and one to that of W. Moreover, in order to provide for the usual variations, known as condensed, standard, and expanded letters, these triangles come in sets of three of different proportions.

33. *Irregular curves.* Irregular curves, Pl. II. Fig. 10, are instruments cut out of thin white wood, pear wood, ebony, etc., or of metal or horn, and are perforated so that the edges of the apertures and the exterior edges, together, afford a great variety of curves of varying curvature. The variety of patterns of these curves is so great that only this general rule can be given as regards choosing them. Select for general use a pattern which has one or more very sharp curves, and one or more curves of large radius. Wooden curves are of two thicknesses; the thicker ones, usually of ebony or other fine material, have generally more smoothly finished edges. Pl. II. Fig. 10, is given as one specimen of the kind of assemblage of curves that it is convenient to have in an irregular curve.

34. *Circular rules* are wooden rulers, cut to fit arcs of circles of various radii. They form, for any given case, a very cheap substitute for beam-compasses, yet, on the whole, are of doubtful utility, as each ruler can serve, except approximately, in drawing circles of only two different radii—or of four, if a circular slot be made in the ruler—the edges of the rulers being concentric arcs.

35. *Pencils.*—The pencils used in geometrical drawing should be of the hardest kind. Small pencils are prepared for more convenient use in the compasses.

The proper mode of sharpening pencils for hand use in geometrical drawing, is to an edge, and not to a point. Those pencils which are attached to small compasses are better when sharpened to a point. Pencils may conveniently be sharpened to an edge by rubbing the opposite sides of the lead upon a fine file. See Pl. I., Fig. 7.

§ c. Scales.

36. The instruments comprised in this section: viz. *scales* of all kinds, are said to be indirectly used, because whether applied to

The paper or not, points and distances are contained by them which are first transferred to paper by compasses, or a needle point, so that lines can be afterwards drawn with other instruments—lines of the required length, through the required points.

36. There are many varieties and forms of scales, which may be grouped, as in the following table:—

SCALES AS TO	Construction, are	Simple, of one kind	Linear, for lines	Single.	Plain.	In form.	Flat. { Surface Scales. Edge Scales.	Triangular.	Graduated throughout to the smallest divisions. Partially graduated to smallest do.
	Superficial, for distances in two directions.	Grouped, of various plain and diagonal scales. Gunter's Scales. Linear Plotting Scales.	In unequal parts.	Offset Plotting Scales.					
					Mixed, of diverse kinds, linear and angular	Ivory. Wood. Metal. Pasteboard. Horn.	Protractor Scales. Sectoral Scales		
	Material, are of								
	</								

In this table, scales of all kinds are analyzed, and reduced to their elements. The table will be best understood by constantly comparing it with the following definitions and explanations of scales in their actual form.

37. The most general definition of the word scale is, a series of gradations—of distance, of value, of weight, of heat, of beauty, of worth, of anything, in short, which may exist in different degrees.

In geometrical drawing, a scale is understood to be a contrivance by which any small distance, as a half inch for example, is taken to represent some, or any one, of the ordinary units of length, as a foot among mechanics, or a chain among surveyors.

38. To understand fully the reason for having scales, we must consider that the ideas of *shape*, and of *size*, are distinct and different.

The *shape* or *form* of an object consists in the *relative positions* of its lines and surfaces. In other words: Form is a determinate

assemblage of points. Its *size* depends on the number of units of size, whether of length, surface, or volume, which it contains.

Now, nearly all the objects of which we make drawings, are so large, that we can make the drawing represent the *shape* only of the object, and not its *size*; but if, for example, we understand that each *inch* on the drawing represents *ten feet* on the object, we shall, by remembering this, get from the drawing a correct idea of the *size*, as well as of the *shape*, of the object drawn.

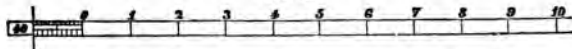
39. *Linear scales* are those in which a certain length, as a rod, on the object, is represented by a certain distance on the scale, as an inch; or conversely, in which a certain distance on the scale "stands for" a certain distance on the object. These two expressions, each of which is the converse of the other, are given because they are brought into use in the notation of the ivory scales, as will be shown presently.

Plain linear scales are those in which the divisions are laid off on a single straight line, as in Figs. 2 and 3. *Diagonal scales*

FIG. 2.



FIG. 3.



embrace a system of parallel lines, divided by parallel diagonals, as in Figs. 4 and 5.

FIG. 4.

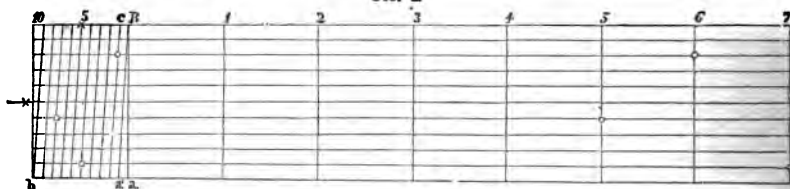
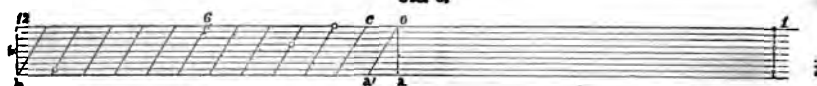


FIG. 5.



40. The *plain linear scales* of equal parts contained on the usual *flat ivory protractor scales* are numbered and named in two ways. On the face of the scale, *i. e.*, the side which is bevelled and

radially divided on three edges, there are a number of scales marked $1\frac{1}{2}$, $1\frac{1}{4}$, 1 , $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, etc. to $\frac{1}{8}$, meaning inches in each case.

These are called scales of $1\frac{1}{2}$ inches to a foot, yard, or mile, or whatever the unit of measure of the object drawn may be; $\frac{3}{4}$ of an inch to the foot, etc.; $\frac{1}{2}$ an inch to the foot, etc. Fig. 2 shows the plain linear scale of $\frac{3}{4}$ of an inch to any unit of measure. Several points about this scale may now be explained, and when understood, all similar scales will be understood.

(a.) If it be used as a scale of $\frac{3}{4}$ of an inch to an *inch*, then each large space, as from 0 to 1, 1 to 2, etc., is considered as an inch, and is called an inch in using the scale.

(b.) The space from 0 to the left is divided at the bottom into twelve parts, and on top into ten parts. In the division into twelfths, the 3*d*, 6*th*, and 9*th* lines are made longer than the others, so as to catch the eye, and assist in readily taking off distances.

(c.) When, therefore, the scale is used as a scale of $\frac{3}{4}$ of an inch to the *foot*, each large space, as from 0 to 1, represents a foot, and is called a foot.

Since twelve inches make a foot, the twelve small divisions at the left represent inches, and are called inches; thus, from the line at 3 to the sixth line at the left of 0, is three feet six inches, *i.e.*, it represents three feet six inches on the object to be drawn.

(d.) If the scale be used as one of $\frac{3}{4}$ of an inch to a *yard* or rod, then from 0 to 1 represents one yard or rod, and the twelfths, not being "aliquot parts" of a yard or rod, represent no "denominate number." The same is true of the tenths just over the twelfths.

(e.) This scale may conveniently be used as a scale of $\frac{3}{4}$ of an inch to 10 feet, yards, rods, etc. In that case, from 0 to 1, 1 to 2, etc., each represents ten feet, rods, etc., and the space to the left of 0 being divided at top into ten equal parts, each of those parts will be one foot, rod, etc. Thus, from 4 to the fourth dot at the left of 0 is 44 feet, rods, etc.

41. Similar remarks apply to all the linear scales found on the face of the ivory scale. Turning therefore to the back of the scale, we find a series of plain scales, marked 20, 30, 40, 45, 50, 60, 80, etc., at the left hand end. These are read as scales of 3 or 30 feet, rods, etc., to the inch, . . . 5 or 50 feet etc., to the inch, as will now be explained. Fig 3, represents the scale marked 40, taken from the back of the ivory scale; now—

(a.) Each of the spaces, as from 0 to 1, etc., *really is* a quarter of an inch, and the one at the left hand of the 0 division is divided

at top into *twelve* equal parts by dots, and at bottom into *ten* equal parts by short vertical lines.

(b.) Then, if this scale be used as a scale of *four* feet to the inch each large space, being one fourth of an inch, will represent and will be called *one* foot; and the left hand space or foot being divided into twelfths, each of those twelfths will represent, and will be called, an inch. Thus, from the figure 9 to the eighth dot at the left of 0, is nine feet eight inches on the scale of four feet to the inch.

(c.) If the same scale be used as a scale of *forty* feet to the inch, then each large division, being $\frac{1}{4}$ of inch, will be $\frac{1}{4}$ of forty feet, or ten feet; and the left hand space, being divided into ten equal parts, each of those parts will be one foot. Thus, from the figure 7 to the fifth of the divisions into tenths, is seventy-five feet on a scale of forty feet to an inch.

(d.) In using the above scale as one of four yards to an inch, the divided unit at the left of 0 should be divided into three, instead of twelve, equal parts, so as to give feet; and in general, *the divided unit of any scale should be divided into as many parts as there are units of the next lower denomination contained in the principal unit of the scale.* The student should thus construct scales of various numbers, of miles, rods, etc., to an inch.

(e.) Practically, an inch is the measure taken to represent any convenient number of units of the same, or some other denomination; but in order to become thoroughly familiar with scales, at least a single example of a different representative unit than an inch should be conceived. Suppose a scale of seven furlongs to a rod. Then, first, lay off a few rods upon the ground; second, divide each rod into seven parts, and the parts so obtained will be furlongs by the supposed scale; third, divide one of the sevenths or furlongs into forty equal parts, since forty rods make one furlong and the small parts so found will be rods on the scale.

42. Scales are expressed fractionally. Thus, a scale of $\frac{3}{4}$ of an inch to a foot is a scale of $\frac{3}{4}$ of an inch to twelve inches, $= \frac{3}{12} = \frac{1}{4}$; *i. e.*, on a drawing made to this scale, each inch in length represents sixteen inches in length on the object. Again, a scale of 4 feet to an inch, is a scale of 48 inches to one inch, $= \frac{1}{48}$; *i. e.*, each linear foot on the object is represented by $\frac{1}{48}$ of a linear foot on the drawing.

43. It will be convenient next to consider the remaining scales of *equal parts*; viz. *diagonal scales.* Diagonal scales of units and decimal parts are common, but similar scales of units and vulgar fractional parts are just as easily made, as will be shown directly.

Fig. 4 shows a diagonal scale of units and decimal parts, such as is used in plotting the maps of surveys, where very small scales are used. Let us suppose the scale to be one of 200 feet to an inch. Each of the larger divisions, as from 0 to 1, etc., being half an inch, will therefore be 100 feet. The half inch at the left of 0 being divided into ten equal parts, each of those parts will be ten feet. Now, the eleven horizontal lines of the scale include *ten* equal spaces, and the space, *ab*, on the lower horizontal line is divided into ten equal parts. A diagonal is then drawn from 0 to *a'*, and then the other parallel diagonals are drawn as in the figure. Now in passing on *0a'*, from 0 to *a'*, we depart from the vertical line *0a*, a distance, *aa'*, equal to one-tenth of *ab*, or *ten feet*; hence in passing from 0 to the intersection of this diagonal with the *first* horizontal line from the top, we depart from *0a*, one-tenth of *aa'*, or one-hundredth of *ab*, or *one foot*. Hence the figure represents a scale from which we can take off units, tenths, and hundredths. Thus, calling the same scale a scale of two feet to the inch, the distance between the small circles on the second line from the top is 6.12 feet; the distance between the little circles on the sixth line below the top is 5.76 feet; and the distance between the circles on the lower line but one, is 7.49 feet. That is, we have this rule: Place one point of the dividers on the vertical line passing through the figure expressing the *units*, and on that horizontal line whose number from the top down is the number of hundredths; then extend the other point as many whole *tenths* to the left of the *first diagonal* as there are *tenths* in the given number, then the space on this horizontal line and between *0a* and the first diagonal to the left will be the number of *hundredths* required. The points of the compasses will thus include the entire required distance.

44. Fig. 5 is a diagonal scale of two inches to the foot, such that eighths of an inch are the smallest divisions that can be taken from it. Rule nine equidistant parallel lines, enclosing eight equal spaces, and draw vertical lines across them two inches apart, as at 0, 1, and 12. Divide the space from 0 to 12, and *ab*, each into twelve equal parts. Draw the diagonal *0a'* and the remaining diagonals parallel to it. 0 12, being one foot, each of its twelve parts is one inch. If, then, in passing over eight spaces, from 0 to *a'*, we depart from *0a* one inch, in going over one space on *0a'*, or from 0 to the first horizontal line below, we shall depart from *0a* one-eighth of an inch. Hence we have a scale from which we can take off feet, inches, and eighths of inches. Thus from 1 to the

circle at the left of 0 is one foot two inches ; between the circles on the third line from the top is one foot three inches and three-eighths ; and between the circles, on the lowest line but one, the distance is one foot ten inches and seven-eighths.

45. These explanations show that the absolute distance between the horizontal lines is of no consequence ; all that is necessary is that these lines should be equidistant. Accordingly, in Fig. 4, they are made somewhat far apart, and in Fig. 5 quite close together. It is also clear that the diagonals might have been made parallel to a line joining *a* and *c*. In this case the horizontal lines would have been counted from the bottom one upwards.

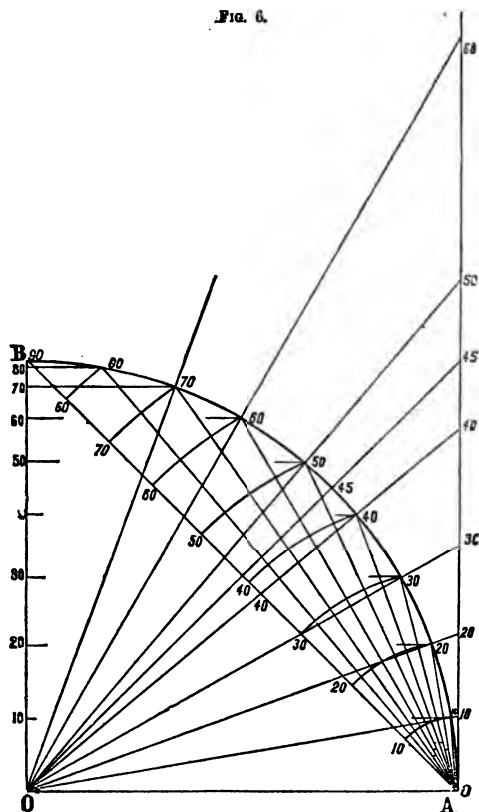
46. Finally, then, we have these *general rules* for the construction of diagonal scales : *First*, for a decimal scale, draw eleven equidistant horizontal lines, and draw vertical lines of division across them to mark the units or larger divisions of the scale ; then divide the left-hand space into ten equal parts at top and bottom, and connect the zero-point at top or bottom with the first point of division at bottom or top, respectively. *Second*, for a diagonal scale of vulgar fractional parts let *N* be the denominator of the fraction expressing the smallest part of the unit of the scale which can be taken from the scale ; let *p* and *q* be factors of *N*, such that *p* is the number of units of the next lower denomination (as inches) contained in a unit of the scale (as a foot). *q* is the denominator of the least fractional part of this unit of lower denomination, which is to be taken from the scale. Rule *q* + 1 equidistant horizontal lines which will include *q* spaces. Divide them by vertical lines into parts which shall be the larger divisions or units of the scale. Divide one of the extreme divisions into *p* parts at top and bottom, and draw the parallel diagonals as described in the first rule. Thus a scale will be formed, such that $\frac{1}{N}$ of the unit of the scale will be the smallest distance which can be taken from the scale. *Let the student construct a diagonal scale of three inches to a foot, from which sixteenths of an inch can be taken.*

47. The linear scales of *unequal parts* are those found mostly on the large two-foot rules called "Gunter's Scales," and used by navigators for working out certain problems in proportion and trigonometry mechanically. Not being much used by the engineer they will be but briefly noticed. The line AB, Fig. 6, is a *scale of chords* formed as follows. Draw a quadrant, AB, and divide it into spaces of ten degrees each. (It would, in practice, be divided into single degrees.) With A as a common centre, describe arcs with the chords A 10, A 20, etc., as radii, till they cut the chord

AB. The spaces on AB, *reckoned from A*, will be the chords of the successive arcs, also reckoned from A. Scales of chords are sometimes seen on drafting-scales. Their use will be shown in *Division II*.

48. If from one extremity of an arc, as 10, Fig. 6, a line be drawn perpendicular to the radius OA, passing through the other extremity, such line is called the sine of the arc. If then, through 10, a line be drawn parallel to OA, the distance, O 10, cut off from the vertical radius OB, will be equal to this sine. Hence, if through all the points of division on the arc AB we draw lines

FIG. 6.

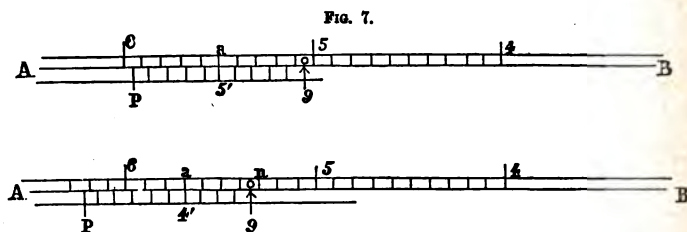


parallel to OA, they will divide OB into a *scale of sines*, the distances from O to the successive points on OB being the sines of the successive arcs reckoned from A.

Again, if a line, AC, be drawn from A perpendicular to OA, and if the radius, O 10, be produced to meet AC, the distance A 10 on AC is called the tangent of the *arc* A 10. Producing the radii through the other points of division on the arc AB till they meet AC, a *scale of tangents* will be formed, the distances on AC reckoned from A being the tangents of the successive arcs, reckoned also from A.

49. Observe that the spaces on the scale of chords are nearly equal, decreasing towards B, that the spaces on the scale of sines decrease somewhat rapidly above 30, and that the divisions on the scale of tangents rapidly increase in size beyond 30. Notice also that the *chord* of 60° equals the radius OA, that the *sine* of 90° equals the same radius, and that the *tangent* of 45° also equals OA.

50. Drafting-scales, as well as other graduated instruments, are sometimes provided with a vernier, a contrivance so named from its inventor, and which is here explained. A vernier is an auxiliary scale attached to a principal one, to allow of the reading of small fractional parts of the least divisions of the latter. Thus, Fig. 7,



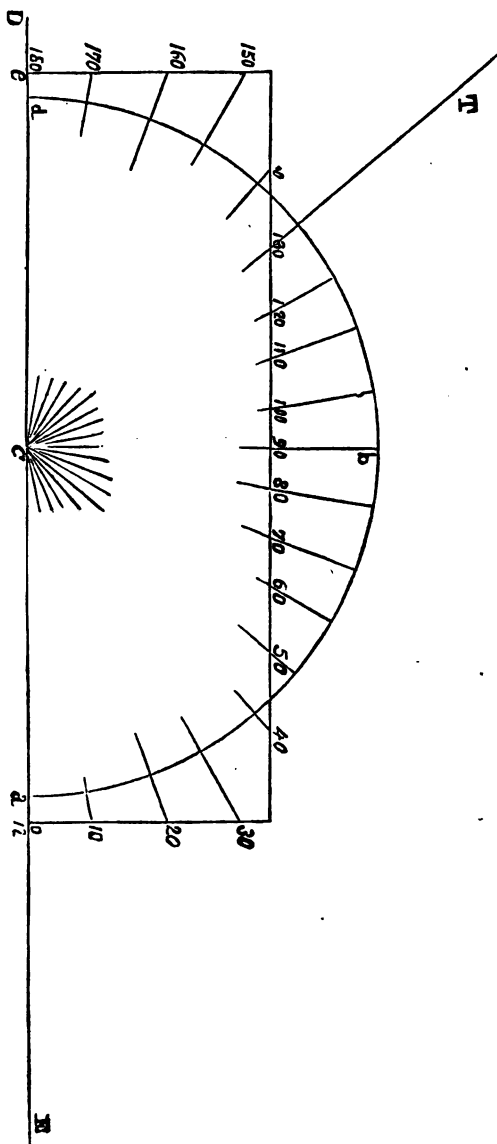
AB is a scale of inches divided into tenths. The vernier, *pg*, slides along this scale, and is divided into *ten* equal parts, which will exactly cover *nine* equal parts of AB. Hence a space on the scale ($=.10$) differs from one on the vernier ($=.09$) by $.01$ of an inch.

Let it now be required to find the distance from the zero-point of AB to *o*. In the first illustration it is a little over 5 inches; in the second, a little over 5.3 inches. In the first case the 5' mark on the vernier coincides with a division of the scale, then 5'o is $.05$ less than the space *a5*, making 5o $=.05$ of an inch, and the required distance 5.05 inches. In the second case the 4' mark on the vernier agrees with a division of the scale, 4'o is $.04$ less than *an*, or *no* $=.04$ of an inch, and the required distance is 5.34 inches.

From these illustrations, we have the following rule for reading the vernier. Set its zero-point at one end of the space to be read, and read on the principal scale up to the division next preceding

this zero-point for the inches and tenths. Then read along the vernier, and the number of spaces there found up to where one of

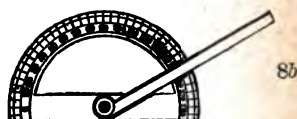
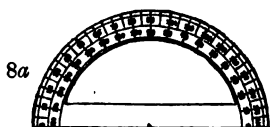
FIG. 8.



its division lines coincides with one on the scale, will be the hundredths.

Other verniers, differently divided and related to the principal scale, can be made on the same principle as the above, in order to give different fractional readings. Let the student construct a vernier to a scale of inches and eighths so as to read to sixty-fourths.

51. *Angular scales* are those used in laying off angles by a direct application of the instrument to the paper. The chief instrument of this kind is that whose construction and use is illustrated by Figs. 8, 8a and 8b, and which is called a protractor. Let C be the



centre of a semi-circle, *abd*, which is divided into 'eighteen equal parts, of 10° each. A flat semi-circular piece of metal or horn, completely graduated to single or half degrees, is a *semi-circular protractor*; and having its centre, C, marked by a fine line or notch, may be used thus to lay out angles. Let CD be any straight line, and suppose it be required to draw a line at the point C, that shall make an angle of 50° with CD; place the protractor with its centre at C, and its diameter on CD; with the needle-point of the pen-handle mark a fine dot on the paper at the line marked 130° ; then through C and this point draw a line, as CT, which will evidently make an angle of 50° with CD. [Let the student exercise himself in drawing lines on both sides of a line, as DE, which shall make given angles with either part of the line, as CD or CE.] A flat circular ring also graduated and used as just described, is called a *circular protractor*.

52. *Full circle protractors*, for the most accurate plotting, are made, 6, 8, and even 10 inches in diameter, and sometimes provided with a vernier, and a radial arm; the former to aid in reading the angle more exactly, the latter, Fig. 8b, to aid in perfectly laying it out on the paper.

Consistency with these refinements, requires that means should be provided for placing the centre of the instrument very exactly upon that point on the paper which is the vertex of the required angle. For this purpose, protractors have been made with a transparent centre, on which are engraved segments of two perpendicular diameters, not intersecting, but limited by a very small circle,

leaving the centre on the paper visible, and easily set by the eye to the centre of the little circle.

Protractors, exceeding a semi-circle by the width of the straight or diameter piece, are made, and hence have their centres on the inner edge of that piece.

Paper protractors, half, or whole circle, are made of drawing-paper, Bristol board, or tracing-paper. The latter are of 14 inches diameter, and divided to quarter degrees. The others are from 4 to 8 inches diameter for half circle, and 13 inches for whole circle instruments, and the larger ones are graduated to half degrees.

53. Suppose, now, that any rectangular piece of ivory or metal were placed with one of its longer sides, *en*, to coincide with DE, and with the middle point, C, of that side coinciding with the centre, C, of the semi-circular protractor. Next, suppose the radial lines of division of the semi-circle to be produced where necessary, so as to intersect the sides of the rectangle. It is evident that since the points of division thus formed are on equidistant radial lines, they may as correctly be used in laying off angles as may the points on the semi-circle, and with the advantage that as *en* is usually greater than *da*, the points, 10–20, etc., of the angles which are quite acute, are further from the centre, C, than the corresponding points of the semi-circle, and may therefore be more accurately located. Further details about protractors are more properly found in works on surveying and mapping.

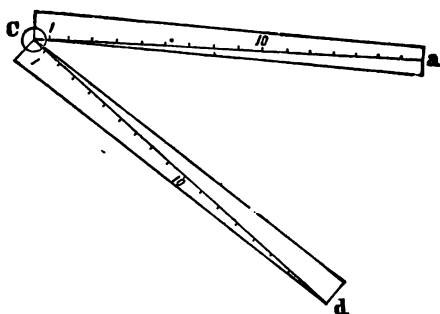
54. *Mixed scales* are those which embrace any two or more of the scales previously described, or other similar scales, both linear and angular. Of these the common six-inch "*protractor scale*" is in very general use among both geometrical and topographical draftsmen. It contains on three of its edges a protractor scale, and on one of its faces a diagonal scale of hundredths, which two scales are much used in plotting the notes of surveys, since field measurements are usually taken in units (chains) and hundredths (links). It also often has on the edge containing the centre, C, of the protractor a scale of equal parts and tenths called an *offset scale*, for plotting the distances from a straight chain-line to an irregular fence or shore-line. These distances are measured at right angles to the principal chain-line, and are plotted in that direction by placing the points C and 90 both on the given line, as DE, in which case the divided edge *en* will evidently be perpendicular to DE.

55. Besides these scales, which are chiefly used by the surveyor, the protractor scale contains on its face (and the bevel-edged side

containing the angular divisions may be called the face), a number of plain linear scales, used by the draftsman, and marked IN. $\frac{1}{2}$, etc., which are scales of inches to the foot, as already explained—see Arts. 37–41—and on the back a number of scales marked 30–40, etc., which are scales of 3 or 30, etc., feet to the inch, as previously explained.

56. Other forms of mixed scales, which need be but briefly noticed, are “Gunter’s Scale,” already mentioned, and the “Sectoral Scale.” These are not described in full, since the first is used chiefly by navigators, and both are designed, by affording scales of logarithms of numbers, and of the common trigonometrical lines, to embrace a sort of mechanical arithmetic—*i.e.* these scales enable one to make out from them, with the help of compasses, mechanical solutions of various problems in finding the unknown from the known parts of triangles. Such solutions are, of course, far less laborious than those which consist of computations; but as they give rather coarse approximations, the instruments used in making them have, at least among engineers, been mostly replaced by the numerous and very full pocket-books of tables now so much in use, and which save a great deal of computation.

FIG. 9.



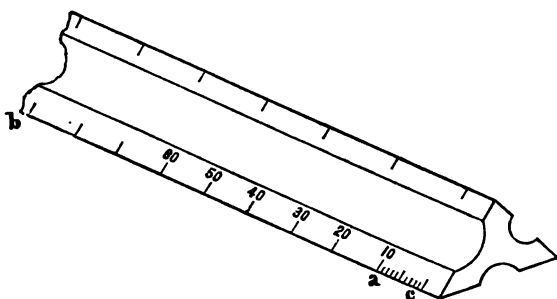
Briefly, then, “Gunter’s Scale” is a solid two-foot rule, usually of box-wood, and brass-bound when best made, containing inch and half-inch diagonal scales; scales of chords, sines, tangents, logarithms, etc., whose uses will be found described in scientific encyclopædias and works on navigation.

57. The “Sectoral Scale” is a double scale, jointed like a carpenter’s rule, and having on each arm, besides scales similar to those of Gunter’s Scale, a linear scale which passes through the centre of the joint, so that any proportional parts of distances can

be taken off, on the familiar principle that parallels to the base of a triangle divide the sides into proportional parts. Thus, in Fig. 9, which is an incomplete sketch of this instrument, let Ca and Cd be two linear scales, passing through the centre, C . Let it be required to divide any distance, say $5\frac{1}{2}$ ins., into tenths, by this instrument. From any scale of inches take five and a half inches in the dividers, and placing one point at the ten-mark on one arm, extend the other point to the ten-point on the other arm; then from 1 to 1 will be one-tenth of the space 10 to 10, *i.e.* of $5\frac{1}{2}$ inches. Many other interesting and curious operations can be performed by the help of this instrument, but being, perhaps, in the present condition of drafting practice, quite as much curiosities as utilities, the description of these operations is left to be sought for in extended treatises on instruments, encyclopædias of arts, etc.

58. A few miscellaneous notes will conclude this description of scales. Scales have been very generally made on broad, flat, thin slips of wood, ivory, or metal, so that distances might, first, be taken from them by compasses, and then, secondly, transferred to the paper before the line to be determined could be completed. Such scales are "*Surface scales.*" Many scales are now so designed as to abbreviate this process, by having the graduations upon the

FIG. 10.



edges, so that the two operations just named can be combined in one by placing the graduated edge of a scale upon an indefinite line and pricking off the required distance. One of the leading forms of these scales is the "*triangular scale,*" Fig. 10, which gives six "*edge scales,*" and by a simple modification, in which each scale should be double of another, two may be arranged on each edge, giving twelve edge scales. Another form is the "*engine-divided*" card-board scale, on which the graduations extend to the edge of the paper. These are "*single*" edge scales, each

paper giving but one, or perhaps on both sides, two scales. *Architectural scales* are *flat edge scales*, giving, therefore, only two or more scales, according to their manner of graduation. As the edges are bevelled, on'y one side can be conveniently used, and hence graduated, giving two or more scales each the double of the next one. Protractors are edge scales. The linear scales on protractor scales are nearly all surface scales, that is, they are not on the edges of the instrument.

59. *In respect to extent of graduation*, triangular and architectural scales may have the smallest divisions carried throughout the whole extent of the instrument; or only one of the larger spaces at the end of each of the scales may be thus divided, as seen in the linear scales on the protractor scale. The advantage of the latter mode is its cheapness; the advantage of the former mode is, that it requires no shifting of the scale in order to set off any series of numbers of feet from a scale—say of 40 feet to an inch—the distances being reckoned from a common point, as *a*, Fig. 10, while it is plain that in a scale divided as shown in Fig. 10, the scale must be shifted for every new distance to be set off from *a*, till that point, as *c*, of *ab* is at *a*, whose distance from *a* (=zero of the scale), added to the number of tens on the other side of (0), makes the given number of feet.

Finally, as to kind of graduation, there are *decimal scales*, graduated in tenths, and *duodecimal scales*, divided into units and twelfths, and therefore adapted to the representation of feet and inches. It is much to be hoped, for more reasons than can be given here, that the latter will never be wholly displaced by the former. Each has its proper uses. Both kinds of graduation are found, as already noted, on the protractor scales.

60. With respect to material, ivory and box-wood scales have the advantages that the lines of graduation are more legible; while on the other hand these lines are finer on metallic scales, but hence also more trying to the eyes. Metallic instruments of all sorts readily tarnish, and thus soil the paper; but this difficulty might be obviated in the triangular scale, by making each of its angles a trifle less than 60° , so that only its edges would touch the paper.

61. The "Plotting scale" consists of two pieces, at right angles to each other, one of which can be clamped in any position. A slit is cut in the other, so that any distance at right angles to the edge of the fixed piece can be set off at any distance from the zero-point of that piece.

§ C.—*Instruments and Materials employed in the Execution of a Drawing.*

62. *Right line pens.* These consist of two thin steel blades, as in Pl. II. Figs. 2, 3, 4, only in the right line pen, Pl. I. Fig. 5, the blades are equally curved and attached to a simple straight handle. Ink is applied with a small brush, or slip of paper, between these blades. One of the blades is hinged to the handle, so that it may be opened for cleaning. This construction is omitted, however, in the cheaper instruments. The blades are connected by a screw, *b*, which regulates the distance between them, and therefore the size of the line which the pen will make.

To ensure firmness of working, the blades are either quite thick, or are pressed apart by a spring, or the one opposite the head of the adjusting screw has a little projection, *c*, cast upon it through which the screw works. The joints are single or double in these, and also in all places where a hinge-joint occurs in the several instruments. The tightness of the joint being the same, it may be supposed that the pressure, being distributed over a larger surface in case of the double joints, they would wear longer. *Double pens* have two pens, connected by a bow-spring, to one handle. They are used for drawing parallel lines, and are also called *road pens*.

63. *Pens for drawing circles.* These have been noticed under the head of bow-compasses, Pl. II. Figs. 2, 3, and 4. They should be quite narrow at the points, so as to follow a curve more readily than a broad point could.

64. The "*mapping-pen*," "*lithographic pen*," and the "*crow-quill*," are very delicate and fine-pointed steel pens, shaped like writing-pens, and used in lettering and in retouching, by hand, portions of the draftsman's work. The lithographic and crow-quill pens are stiffer than the mapping-pen. The crow-quill is stiffest of all, and is a substitute for the real crow-quill, which is a very delicate but stiff quill for draftsmen's use. See Pl. I., Fig. 11.

65. *Colors.* The colors necessarily and often used in geometrical drawing are Indian-ink, Prussian blue, Burnt sienna, and Carmine or Crimson lake. Those, some or all of which it is occasionally convenient to have, are Black, White, Gamboge, or Yellow Chrome, Vermilion, Dragon's Blood, and Cobalt.

66. The best *Indian-ink* for tinting and shading is fine grained, of a deep brown rather than a black or bluish tint, of a rounded or shell-like fracture, and is golden-hued when freshly broken. It is musk or camphor-scented, and prepared in highly polished or

gilded cakes, which are ornamented with *finely executed* Chinese characters or other designs. Good ink for lines only is perfectly black.

Cheap kinds of ink have a coarse, dull, black surface, are coarse-grained, and have a black or dirty blue-black tint, are usually not scented, and are coarsely ornamented. India ink, and all the other colors, will crumble if left wet, hence they should be wiped dry after use.

India-ink, being not black, but a very deep drab (yellow and black), is made to appear darker by the admixture of a minute portion of blue, which changes it to an "invisible green." It appears darker when thus modified, because while yellow is the lightest of the primary colors, blue is the darkest.

67. *Prussian blue* is a clear and nearly pure dark-blue, used in various ways hereafter to be mentioned.

Burnt sienna is prepared from a kind of earth, and is a bright reddish-brown.

Carmine is a gorgeous red, slightly inclined to purple. *Crimson lake*, named as a cheaper substitute, is a little more purple, and a little less clear and brilliant.

68. Of the occasionally convenient colors, *Black* may first be named. Two kinds are in common use: Lamp black, made from the soot of resinous substances; and Ivory black, from the smoke of burnt ivory chippings. The latter is, perhaps, a little finer.

69. The best white is *Chinese white*, since its quality of color or "tone" changes but little in drying; while other whites, as "constant white," are like whitewash, appearing almost of an ashes color when laid on wet, and then drying up to a thick white, which covers the tints below with a chalky look.

70. *Gamboge*, a cheap natural product, which can be used as sold by druggists, and *Chrome Yellow*, are light bright yellows, useful in representing brass. So also is *King's Yellow*.

71. *Vermilion* is a brilliant red, slightly inclined to scarlet (yellowish-red), useful to mix with yellow in making brick color; but so heavy as to be held in solution with some difficulty.

72. *Dragon's blood* is a dark and rather dingy reddish-brown, which might serve in representing leather.

73. *Cobalt* is a clear, brilliant, and beautiful sky-blue; preferable, in some cases, to light tints of Prussian blue.

74. All these colors are, when used for drawing lines, applied with the drawing-pen, and are inserted between its blades with a brush. The ink thus used should be about as thick as oil paint, or thick enough to appear of a deep black when the lines become

dry. Trial lines should be drawn on the margin of the paper to determine this.

75. Colors having a high reputation are made by Newman, and by Winsor and Newton of London—made in whole and in half cakes. The latter are of as good quality as the former, and, for the draftsman, are preferable, since he uses them so very much more slowly than the water-color artist does.

76. Besides the colors which come in hard cakes, there are those called *moist colors*, so prepared as to be permanently moist. These come in little porcelain trays in a tin box with folding palettes for mixing. Both the dry or hard, and the moist colors are called *water colors*, in distinction from oil colors, because they are mixed with water; for containing which, bird-cage vessels, or similar small cups, of chinaware or glass, are convenient.

77. While India-ink and water colors can be ground in a few drops of water on cup-plates, or even on bits of broken china, yet articles expressly for the purpose are more convenient. These are of many forms, generally called *ink-tiles* or *palettes*, one of which is shown in Pl. I., Fig. 9. This has a "slope," and three hemispherical "wells." Other forms have three or more transverse slopes; others are thin, with shallow square compartments, and are better adapted for using the colors in a damp, rather than in a liquid state.

78. A different, but very convenient arrangement is a "*nest of saucers*," forming, when put together, a cylindrical pile of shallow cups, about $2\frac{1}{2}$ or 3 inches in diameter. These, when placed inside of a close box in a cool place, will hold the liquid tints twelve hours, more or less, without drying up, and thus will save much waste of color, and loss of time in frequent remixing.

Still again, there are combined water-cups and tiles, the latter circular, with radial slopes outside of the central cup.

79. Colors, including ink, when applied in diluted tints, are spread by a brush. Two kinds of brushes, as regards material, nave, each, their appropriate uses, viz. camel's hair and sable. The latter kind of brush is more elastic and carries a firmer point, and is used where fine strokes, definite in position, are to be made. A brush of camel's hair is finer and softer, and is convenient for laying on flat tints over large surfaces. The larger brushes of this kind have as sharp a point when wet as the smaller ones, with the advantage that they afford a sort of reservoir from which to supply the point with ink, thus preventing those irregularities of tint which arise from the brush becoming dry in the midst of an operation.

80. It is desirable to have a brush called a *softener*, consisting of two large camel's-hair brushes at opposite ends of the same handle—one of them being for use with clear water only, in softening the edges of tints, when shading; then one or two small camel's-hair brushes for small work, feeding the pens, etc.,—brushes of “goosequill” size; and one or two sable brushes of the same size and on long handles. Indeed, there are two kinds of sable brushes, the black, and the red haired, but the latter are much cheaper, and either will answer the purposes of the draftsman.

§ D.—*Articles for the Correction and Preservation of Drawings and Instruments.*

81. These articles are, India-rubber, the erasing-knife, rubber ink-eraser, the fine sponge, hone, screw driver, stale bread, pencil-sharpeners, drawing-board cases, and the horn centre.

All instruments should be put away, after use, perfectly *clean and dry*. For this purpose, a piece of wash-leather, or soft rag, should be kept at hand.

82. If a drawing could be at once located to the best advantage on the paper, and surely made without mistake and with all its lines correctly limited when first drawn, it might be made in ink directly on the blank paper. To avoid the errors inevitable in the first copy of any production, even when made by those most practised, drawings are first pencilled and then inked. The whole theory of pencilling, then, is, to lay out *correct tracks* on which the pen is to move, leaving the mind, during the inking, free from all thought of *accuracy of the construction*, that it may be given to *excellence in execution*. Therefore, the whole of the pencil-construction should be *most accurately* made in the *finest faint* lines with a *hard* pencil.

But even with this division of labor, mistakes will sometimes occur. Remedies for some of them will now be described:

83. *Errors in pencilling* are removed by India-rubber. The rubber should be entirely free from sand. With this condition the “vulcanized” rubber is the most agreeable to use, being effectual and free from stickiness. The use of rubber should, however, be avoided, as it roughens the paper, if it does not make it greasy, and thus hinder the making of smooth ink-lines.

84. Errors in extending ink-lines beyond their proper termination are removed by a sharp erasing knife, which has a short, sharp triangular blade fastened to a wooden or ivory handle. In using

this, to take out a thick line, scrape in one direction with moderate pressure; otherwise there is danger of tearing out the paper. Small, thin blots may be treated in the same way.

A compound of vulcanized rubber, and finely ground glass, called *Green's ink-eraser*, is often preferable to the knife. It may also be used to partially erase tints that are too dark. An article called glass paper, similar to the finest sand-paper, answers the same purpose.

85. Broad blots of any light tint, or black blots, discovered before they are dry, or tints of a wrong color, can, if not in the midst of a considerable quantity of fine work, be nicely washed out with a sponge wet with clean water and gently applied; once to take off the heaviest part of the blot, and again with pure water to prevent a stain from remaining throughout the washed portion. A portion of a cake of India-rubber, rubbed down to a wedge form, is very convenient in erasing any one of a number of lines which are near together.

A light ink line may be erased by dampening it slightly with a moist brush and then rubbing over it with the wedge-shaped rubber, or ink-eraser.

86. Whenever the surface of the paper is made rough by scratching, it should be rubbed down with some hard and perfectly clean rounded instrument, as the erasing knife-handle or the little knob on the pen-handle, before having lines or tints again placed upon it.

So much for general faults, such as often occur. The treatment of errors connected with particular operations will be spoken of with those operations.

It is possible even to erase a portion of the *width* of a line, when that line has been made too heavy. Thus: Cover the portion of the line which is to be retained, with a sharp-edged ruler, held down tightly, and then with a sharp-pointed blade erase the uncovered portion of the line. With a little care this can be done quite neatly.

87. While, therefore, "Finish a drawing without any error or defect," should be the draftsman's best motto, he should never be in haste to reject a damaged drawing, but should exercise his ingenuity to see how far injuries done to it may be remedied. "Never lose a drawing once begun," should be his second motto; and once more, in this connection, since prevention is easier and better than cure, let him always work calmly, always inspect all instruments, hands, and sleeves, that may touch a drawing,

before commencing an operation; let the paper, instruments, and person *be kept clean*, and, when considerable time is to be spent upon a portion of the paper, let the remainder be covered with waste paper, pasted to one edge of the board.

88. For the final cleaning of the drawing, stale bread, or the old-fashioned black India-rubber, if not sticky, is good; but, aside from the carelessness of ever allowing a drawing to get very dirty any fine drawing will be injured, more or less, by *any* means of removing a considerable quantity of dirt from it.

89. Another excellent means of preventing injuries, which should be adopted when the drawing is worked upon only at intervals, is to enclose the board, when not in use, in a bag of enamelled cloth or other fine material.

90. A screw-driver, see *gf*, Pl. I., Fig. 1, is provided for tightening the joints of compasses. A fine oil-stone (hone) is useful for dressing the points of the drawing-pens when they are of unequal length or outline. The best pencil-sharpener is a fine file, on which pencils can be sharpened to an edge, which will last longer than a point. Sand-paper is sometimes used for the same purpose; also a little hollow, conical instrument, with a cutter acting—though with a circular motion—on the principle of a carpenter's plane.

91. Finally, when many concentric circles have to be drawn, the *horn centre* is a convenient little contrivance for preventing the wear of a large hole at the common centre. It consists of a small circular plate of transparent horn-bound with metal. The metallic edge carries three short, sharp points, which give it a hold upon the paper.

Another contrivance, having the same object, is opaque, being wholly metallic. Two diameters at right angles to each other are engraved upon it, which are placed to coincide with lines at right angles to each other through the given centre on the paper. Thus the centre of the instrument is made to coincide with the hidden centre on the drawing.

§ E.—*Special or Higher Instruments.*

92. *Of the higher drawing instruments*, as they may be called, the professional draftsman will learn from instrument makers, or treatises on instruments, while the student has, at most, only occasion to know of their existence.

The following may here be mentioned merely. Descriptions

of them and of several others will be found in larger, or special treatises.*

Centrogaphs, for drawing arcs of large circles.

Elliptographs or *elliptic compasses*, for drawing ellipses.

Pantographs, and *Eidiographs*, for tracing copies on a reduced scale, by continuous movement; not by construction, as when proportional dividers are used.

The centrolinead, a mechanical substitute for the geometrical construction of lines having the same "vanishing point" (distant point of meeting) in a perspective drawing.

The excentric rule, for drawing slightly converging lines, as the sides of the tapering arms of mill wheels.

The opisometer, a wheel contrivance for measuring the length of any curved lines.

Border pens, with three blades, for ruling at once very heavy border lines.

Curve pens. In these the pen is at the extremity of a short horizontal arm, extending from the base of the handle, the design being to prevent the fingers from hiding the work.

93. *Covered lead weights*, for holding open drawings that have been rolled up, or for holding tracing cloth, or paper, or a long ruler, in place; and *prepared ox-gall*, to prevent slightly greasy spots in the paper from producing spotty tints, are among the occasionally useful articles for drafting offices.

* See Stanley's Mathematical drawing instruments.

CHAPTER II.

PREPARATION AND TESTS OF INSTRUMENTS AND MATERIALS.

Preliminary Remarks.

24. All drawing embraces two distinct departments—*Construction* and *Execution*.

The construction of a drawing embraces the exact location upon the paper, with pencil and dividers, of all its points and lines. The execution includes the inking of the lines and the tinting or shading of the surfaces represented.

The chief feature of the *construction* should be *accuracy*; and of the *execution*, *elegance*.

Hence the error already manifestly appears, of pencilling a drawing carelessly, with a view to correcting the errors made, when inking. During the latter process the mind should be free from all need of thought for the *accuracy of the construction*, and should be wholly given to securing *elegance of execution*.

The essentials to *accuracy* are fineness and exact location of lines. The essentials of *elegance* are black, firm, smooth *lines*, uniform when meant to be so; smooth, clear, and uniform *tints*, not spotty or ragged; and evenly graded *shading*.

It is now obvious enough that, to avoid waste of time and labor previously spent, through spoiling a more or less elaborate *construction* by bad *execution*, thorough training in the elementary *processes* of construction and execution should precede their application to the drawing of complicated diagrams and complex objects.

Preparation of Paper.

95. *First*.—In fastening paper to the board with thumb-tacks, it is well to place under the head of the tack a washer of soft leather, or several folds of soft paper, since the friction between the washer and the sheet of paper—due to the steady and uniformly distributed pressure of the tack—will aid in holding the paper, and will thus tend to prevent the tearing produced by

pulling the paper against the bare shank of the tack. This, though really a small matter, is worth observing for convenience' sake in making plates of line drawings, which only need pinning down. Gum, lip-glue, sealing-wax, wafers, or paste, may be used in place of the thumb-tacks.

Second.—To secure paper by damp stretching.—Provide some dissolved gum-arabic—or better, some of the prepared mucilage now everywhere sold in convenient bottles, together with a stiff bristle-brush, by means of which it may be conveniently applied to the paper. Also have at hand a small soft sponge and some clean water. Good mucilage is very thick, of light color, and agreeably preserved by oil of clove, in preference to the nauseous carbolic acid or like substances sometimes used.

Hold the paper towards the light, so as to read the manufacturer's name, which is marked in water-lines on each sheet. The side towards the eye, when the name can be read, is considered as the right side or face of the paper, it being supposed to be more free from imperfections than the back of the sheet is. Lay the sheet with the back upon the board, and wet the face well—except for a space of about half an inch in width all around the edges—by carefully stroking the surface with a sponge full of clean water. When the sheet is wet enough, a little water will trickle from one corner if it be held up by the opposite corner.

Before or after wetting the sheet, it makes little difference which, fold over the *opposite edges* in succession, for a reason which will soon appear, making the fold about half an inch wide. Essentially more than this is a waste of paper; much less will hardly be sufficient. With the bristle-brush apply thick gum-arabic or mucilage quickly to the folded edges, and then immediately, or before the paper becomes dry, fold back the edges upon the board, rubbing them down briskly, so that the gum will “set.” This last point is of more consequence in the case of new boards. These edges should be folded over on opposite sides of the sheet in succession, as this mode tends to stretch the paper uniformly and tightly, without ripping up parts previously fastened or bringing out a bunch of wrinkles in one corner, both of which things are liable to happen if the folding be made in direct order around the sheet. The paper, thus folded down, must be left to dry in a cool place, and watched, to see that it does not peel up. If dried by a fire, the paper shrinks with force sufficient to tear up the edges before the gum has “set.”

96. When many successive sheets are to be fastened to the same

board, the old fastening should be thoroughly scoured off before a new sheet is mounted, since, if the two sheets are not put in precisely the same position, the new sheet will adhere to the gum left by the former one, in spots which may be within the border of the drawing on the new sheet, much to the vexation of the draftsman.

The wetting of the paper on either the face or back exclusively is a matter of little moment. It dries sooner when wet upon the face, and is perhaps less likely to be stained by the board.

Successive sheets can be fastened to the same board without removing the preceding ones, if these contain drawings in ink only, and in any case by putting sheets of glazed paper between the successive sheets of drawing-paper. This is perhaps all that need be said here. As the young draftsman progresses he will learn by experience whatever minor points might be here mentioned.

97. *Third.* To fasten paper into the framed board, merely wet it and button it smoothly and tightly into the frame. Though this process seems very simple, yet boards of this kind do not seem to be in common use, probably because they will serve for sheets of but a single size.

98. *Fourth.* To mount paper on cloth.—This process it is often convenient to understand, as large drawings, which are to be subjected to considerable wear, should always be thus mounted. Let the cloth—white cotton or linen is used—be stretched to the utmost upon a frame or upon a floor, wooden wall, or large board, and fastened with numerous tacks. Then, having some strong paste, apply it to the back of the paper with a large brush, carefully removing all lumps, etc. Next, lay the paper evenly upon the cloth, and, having hands perfectly clean, commence at the centre of the sheet, and with the palms of the hands spread out the sheet smoothly and tightly, and afterwards go over it with a patting motion with a cloth, being careful to rub it down smartly on the edges. Leave the whole till it becomes dry, when it may be taken up and the edge may be bound with silk.

99. *Fifth.* To mount paper on pasteboard.—The paper should be allowed to expand with the moisture of the paste before being applied to the pasteboard. To prevent the shrinking of the paper from warping the board, waste paper should be pasted to the other side of the board.

Preparation of Indian-Ink.

100. Although cheap colors are largely used as toys, yet here

and there some one is found not familiar with the mode of preparing water-colors for use. The end of a cake is touched to some clean water, and then ground upon a porcelain or marble tile. Water, rather than oil, being thus used as a solvent, the colors are called water colors.

101. To prevent crumbs from scaling off in rubbing the colors they may be ground upon the tip of the finger, or on a bit of sponge, and from those transferred to the tile.

When thus mixed, the ink should be put upon a dry tile, and diluted with water before it becomes dry; for if a brush full of water be stirred about upon a tile covered with dry ink, or if the ink be rubbed from the finger into a well of the tile containing water, it will almost invariably form into scales which cannot be broken up. These remarks apply to ink mixed in the wells of the tile and diluted to pale tints. For drawing lines, the ink should be ground vigorously in the long channel of the tile till it is black and quite thick.

102. Time and ink are often wasted by preparing much more than is needed for use. No more should be prepared than is wanted for immediate use; freshly rubbed ink being better than stale ink. Nests of ink-saucers, with ground rims, will, however, preserve the ink in a fluid state for several days.

When ink that has dried on the tile is to be reground, it may be done by grinding fresh ink into it, using only a drop or two of water, or by a brush very slightly damp, which will prevent the ink from peeling up in troublesome scales.

Processes in Testing Instruments.

103. In this section we shall notice the tests of the several instruments, considering them in groups, as in the chapter on the description of instruments.

First.—To test the Drawing-board. The edges of the board should be perfectly straight. It is a matter of comparative indifference whether its angles are truly right angles or not, as will be evident after the uses of the T square and triangle shall have been further explained. Since the board is rigid, its edges can be sufficiently tested by sighting along them. Or a strip of board whose edge has been planed smooth may be applied edgewise to the edge of the drawing-board, when, if that edge be convex, the strip will roll upon it, and if it be concave the space between it and the strip can be seen by holding both up to the light.

The flatness of the board can be sufficiently tested by a straight edge, or by observing whether it will lie steadily upon a flat table

104. *Second.*—*To test the joints of the compasses.* The test for a uniform tightness of the joint has been given, but sometimes the pivot appears to be slightly smaller than the holes in the pieces of the joint through which it passes, as shown by a jerk or play when the motion of the points is reversed, which play is not remedied by tightening the joint, but can be prevented only by plugging up the joint, if possible, around the axis, or by having a new pivot made. Hot wax can be used for plugging.

105. Another important point in connection with compasses is the position of the needle and other points, particularly in compasses adjusted to describe very small circles. In this case the plane of the legs should be exactly perpendicular to the joints at the head and at the insertion of the movable legs. If, on trial, defects are discovered in these respects, they may be removed by filing the heads of the movable legs at their insertion. As it is impossible to draw very small circles neatly if the compass-legs are twisted, care should be taken to get instruments having firm joints, and to use them carefully.

106. *Third.*—*To test the straightness of the edges of the T square.* With an eye well trained, perhaps no way is better than to sight along its edge, as mechanics do to test the straightness of the edge of a board.

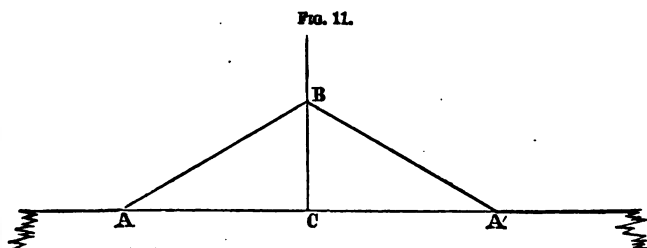
107. Another very good means of testing is as follows: Insert two fine needles in a flat sheet of paper, or into the surface of a smooth, hard board, place the blade of the T square with its edge against the needles, and with a sharp pencil—or a fine steel-point in the case of the board—draw a fine line directly against the edge of the T square. Change the blade end for end, keeping the same edge against the needles, and redraw the line. If the two lines coincide the edge is perfect. If not, the separation of the lines will show where the imperfections of the blade exist.

108. A third test is to place any straight ruler side by side with the blade of the T square, with their edges pressed together, and to hold them up to the light in this position, when, if both edges are perfect, no light can be seen between them when the eye is carried back and forth over the junction of the edges. The straightness of the edges of the triangles can be tested in the same way, if they are not too thin.

109. *Fourth.*—*To test the parallelism of the edges of the blade of the T square.* Draw any straight line of considerable length

on a flat sheet of paper or on the board, and by a geometrical construction, according to any of the usual methods in Elementary Geometry, lay out very accurately two lines perpendicular to the line already drawn, and at a distance apart equal to the width of the blade of the T square at any one point. Then if the edges of the blade are parallel, they will coincide with these lines.*

110. *Fifth.*—To test the angles of the triangles. Fig. 11. To test the right angle of either triangle place a side, as AC, adjacent



to the right angle, against the edge of the T rule, C, and draw a fine line against the other side, CB, of the right angle; then reverse the triangle, placing it in the position ABC, and see if the line BC, drawn with the triangle in this new position, coincides with the line BC, as first drawn. If, instead of coinciding as in the figure, the lines BC *diverge* in going from the edge of the T rule, C being the position of the vertex of the right angle in both positions, the supposed right angle is a little acute; if the same lines *converge*, the triangles being in contact at B, as they depart from the edge of the T rule, the angle in question is obtuse.

In the 45° triangle the two acute angles should each equal half of a right angle.

In the 30° triangle two angles drawn side by side with the angle of 30° should just equal the other angle, which is an angle of 60° .

111. *Sixth.*—To find whether the blade of the T square is truly at right angles with the head. By any of the familiar methods construct, on a large scale, a line perpendicular to the edge of the board, then place the T rule with its head against the edge of the board, and see if the edge of the blade will coincide with the line thus drawn. Or having, as just described, tested the triangles and found them true, place one of the sides adjacent to a right

* NOTE.—Another test will be given after the use of the T square has been more particularly described

angle against the head of the T rule, and see if the other adjacent side will coincide with the edge of the blade. If the want of coincidence is at the acute angle of the triangle the blade makes an obtuse angle with the head on the edge which is tested. If the lack of coincidence be at the right angle, the edge operated upon makes an acute angle with the head on the side tested.

112. *Seventh.*—*To test the graduations of a scale.* The tests for the accuracy of scales will be obvious enough when the student shall have arrived at that stage of his course at which scales need to be often used.

It is enough to observe here that for all scales of *equal* parts, whether "linear" or "angular," it is sufficient to see if any distance, as indicated on one part of the scale, will be equal to the same distance as taken off from any other part of the same scale. If any essential error is found, especially in the scales most frequently used, the instrument should be exchanged.

113. *Eighth.*—*To test the drawing-pens.* These should mark well when held vertically, and make finer and finer lines as the blades are closed by the screw. Otherwise, the pen *may* be used in any position in which it will mark evenly and smoothly, keeping it parallel to itself; but the compass-pens must be made to mark well when vertical, for thus only can they be held.

To dress a drawing-pen, procure a small fine oil-stone, and some fine oil—close the blades by their adjusting-screw, and draw the pen very gently upon the stone, in a plane perpendicular to the stone, to give coincident outlines to the points. Then, in connection with frequent trials of the pen (first entirely freeing it from oil), grind the outer faces of the blades very slightly with a pushing motion, to make them of the proper thinness. No exact description of the process can be written, but the above may save the learner from ruining his pen by too severe, or too much grinding.

It is essential that the joints be firm, so that the blades will not slide upon each other; also that the blades be not so sharp as to cut the paper, or touch above the points, on the inside.

114. *Ninth.*—*To test the point of a new brush.* Agitate it well in a cup of water to expel the air from among its hairs, then bring it to a point by the lips, and see if the point holds while the brush is gently manipulated upon any smooth surface. If it splits up into two or three points, it may perhaps be improved by cutting or pulling out a few of the interior filaments.

DIVISION II.

FUNDAMENTAL OPERATIONS.

CHAPTER I.

OPERATIONS IN CONSTRUCTION.

115. The instruments being all put in the best order before commencing work, the careful draftsman will make it a point to keep them in perfect adjustment for the longest possible time.

116. *First.*—To draw lines with the T rule. Place the instrument with its head against the left-hand edge of the drawing-board, and holding it in that position—pressing against the middle of the head, to prevent slipping or turning—draw the line steadily with the pencil or pen lightly pressed against the blade. By sliding the instrument along the edge of the board, lines may be drawn parallel to the first line. Hence, if the edges of the blade are parallel, lines drawn by pressing the pencil against both of them will be parallel. [See note, p. 43.]

117. *Second.*—To draw lines perpendicular to those drawn with the T rule as just described. This could be done by placing the head of the T rule against the adjacent side of the board, but as the right angles of the triangles are more to be depended upon than those of the board, the following way is better. Place the triangle, with a side adjacent to a right angle, against the blade of the T rule, and against the other adjacent side draw the required line. By sliding the triangle along, parallels can be drawn, which will all be perpendicular to the lines drawn by the T rule.

118. *Third.*—To draw all possible radial lines through a given point with the T rule and both triangles, each triangle being used separately with the T rule. 1st. A line can be drawn through the given point with the T rule, and parallel to the length of the drawing-board. 2d. A line perpendicular to this can be drawn as described in (117). 3d. Place the 45° triangle with its hypothe

nuse against the edge of the T rule, and bring each of the other sides, in turn, to pass through the given point, and draw two lines. Thus the centre lines of the arms of an eight-armed wheel would be found. *4th.* Place the hypotenuse of the 30° triangle against the edge of the T rule, and, as above, bring each of the other sides to pass through the given point, and draw two lines. *5th.* Reverse this triangle, still keeping the hypotenuse against the same edge of the T rule, and then draw two lines. Rejecting the 45° lines, the remaining ones will be the centre lines of the arms of a twelve-armed wheel. The student will readily see which lines would form the centre lines for a six-armed wheel.

119. *Fourth.*—Through a given point, to draw a parallel to a given line. Pl. I., Fig. 16. Let u be the given point, and pr the given line. *1st.* Place any side of either triangle upon the given line, as seen by the upper position of the triangle C. *2d.* Bring up the other triangle, B, or a ruler, against the triangle C, to serve as a rest, or guide. *3d.* Slide the triangle C upon B, till the side placed against pr passes through u ; when the required line us , parallel to pr , can be drawn.

When the hypotenuse of C is placed against the given line, as in Fig. 15, a longer parallel can be drawn.

120. *Fifth.*—Through a given point, to draw a perpendicular to a given line. This can be done with the two triangles, one fixed and one movable, in three ways: *first*, by simply shifting the movable triangle; *second*, by turning and shifting it; *third*, by turning it in space.

First, by shifting. See Pl. I., Fig. 16. Let us be the given line and u a point through which a perpendicular to us is to be drawn. Place either side of the right angle of one of the triangles, as C,

FIG. 12.

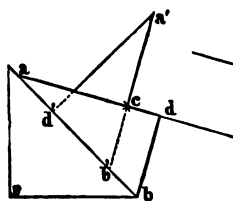
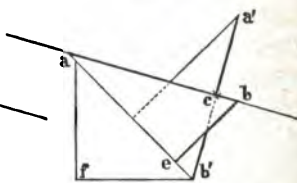


FIG. 13.



against the given line us , and then slide C upon the fixed triangle B (on which it rests as in drawing parallels) until the other side of the right angle of C passes through the given point, when the desired perpendicular, ut , can be drawn.

Second, by turning. See Fig. 13. Placing the *hypotenuse* of the moving triangle, abc , against the given line ab , turn it one quarter round, about the square corner c , when it will take a position similar to $a'b'c'$ from which it can be slid on the fixed triangle afb' , until the hypotenuse $a'b'$, now perpendicular to its former position, shall pass, as shown, through the given point c' .

Third, by turning in space. See Fig. 12, where c is the given point, and ad the given line. The movable triangle being found in the position adb , take it up (keeping abe fixed) and lay it down in the position $a'd'b'$, when the hypotenuse $a'b'$ will be perpendicular to the former base, ad .

By either the second, or the third methods, a longer perpendicular can be drawn, than by the first.

A ruler can of course be substituted for the fixed triangle.

121. *Sixth.*—To take off a distance from a linear plain or diagonal scale. The points of the scale to which the dividers must be applied, in order to take up a certain distance, have been mentioned (Art. 43). It only remains to describe the proper handling of the instruments. Do *not* let the scale lie on the table, and take the dividers in both hands, as the scale will then slip about; but hold the scale firmly in the left hand, and apply the compasses with the right hand, disposing the fingers so as to control both points. The compasses should be held in a plane perpendicular to the scale to prevent slipping, and when removed should be held by the joint, so as to avoid all pressure upon their legs tending to close them. The direction about holding the scale will not apply in using the spacing-dividers; hence, when they are used, it is better to have the scale constructed on the sheet of drawing-paper which is firmly fastened.

122. In the case of any edge scales, as the triangular scale, remember that unless divided throughout, the distances on them are reckoned each way from the 0 point, a , Fig. 10; hence, at the given point on any line along which a distance is to be laid off, as c , place the mark which, reckoned from 0, gives the inches (7 ins) of the distance to be laid off, and, with the needle-point, make a puncture, as at d , at the figure denoting the feet (2) in the same distance. If ab were in 10 parts, each representing a foot, read *units for inches and tens for feet* in the explanation just given; or read feet and tenths, etc. (59.)

123. *Seventh.*—To construct an angle by means of the scale of chords. A chord may be taken as the representative of an arc an arc may be considered as the measure of the angle at the

centre, included between the extreme radii of that arc, and the chord of 60° is equal to the radius of the circle, hence—see Fig. 14—to lay out an angle of 30° , draw an indefinite arc, ab , with the chord of 60° from Fig. 6, as a radius, then take the chord of 30° and apply it to the indefinite arc, and join the points a and b , thus found, with the centre C , and aCb will be an angle of 30° . Proceed likewise for any other angle.

In addition to the explanations naturally given in (51), the following may be noted :

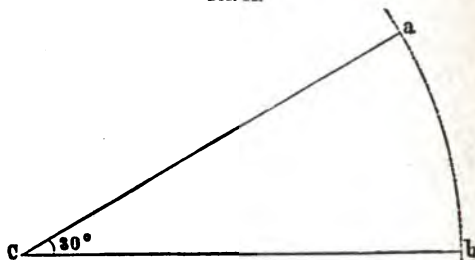
1°. To apply the diameter of the protractor very exactly to a fine pencil line, which may be obscured by the edge of the instrument, it may be useful to make two fine black dots on the line, with a sharp pencil point.

2°. To avoid the glare of the metallic surface, which may obscure its graduations, in many lights, the instrument may be shaded by the hand.

3°. Highly desirable as it is to possess eyesight capable of following the divisions exactly, yet, especially in case of protractors divided to half degrees, it may be best to use an eye-glass in perfectly transferring any division to the paper.

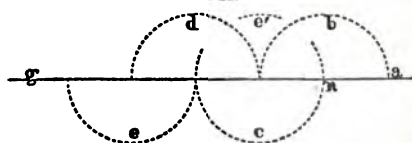
124. *Eighth.*—To operate the compasses in setting off a distance,

FIG. 14.



as *an*, several times on a line. Do *not* hold the instrument in both hands, and take up both points from the paper every time the dis-

FIG. 15.



tance is to be laid off. Also, do *not*, when turning the compass in

one hand, so as to step off the distance, allow the moving-point to describe semicircles on one side only of the line, as at *b*, *e'*, and *d*, on the line *ag*, Fig. 15, for thus the fingers will become entangled at the head of the dividers; but, holding the instrument by the joint, between the thumb and forefinger, keep some one point of the dividers always on the line, and make the other point describe semicircles alternately on opposite sides of *ag*, as at *b*, *c*, *d*, and *e*.

125. *Ninth*.—To operate the compasses in drawing ordinary and very large or very small circles. In drawing ordinary circles, hold the dividers as in setting off distances, and carry them steadily around, keeping a slight pressure on the fixed leg, which should also be a little longer than the other one, and keeping both the needle and pencil-points vertical. In using beam-compasses to draw large circles, steady the needle-point with one hand and carry the pen or pencil-point around with the other hand. The drawing of very small circles with the bow-compasses, is an art which must be acquired mainly by practice with a good instrument. It is one of those things on which the draftsman can profitably exercise himself at odd moments. The ink, while black, should be thin enough to flow freely, the pen-points should bear evenly upon the paper, the fixed leg should be the longer one, and the instrument should be quickly and lightly turned.

The following direction is of especial importance to beginners :

In setting the compasses to the radius of a given pencilled circle, just before inking it, it is necessary that the compass-points should exactly include the radius, *after* all pressure used in setting them to the radius has been removed. Otherwise they may spring slightly out of position, giving a circle slightly greater or less than desired; or, if the pressure used in setting the points be removed while inking the circle, it will not return into itself exactly.

126. *Tenth*.—Of the manner of using pencils. As stated in Art. 82, the theory of pencilling lies in the fact that it is generally impossible to do anything perfectly at the first trial. Since, then, mistakes will be made, it is better to make them in alterable pencil lines than in comparatively unalterable ink lines. Therefore the draftsman should never hurry through with the pencilling, intending to make the inking correct, but he should always make the pencilled constructions perfect, so that their lines being merely paths for the pen to go over, the whole attention may be given, at the time of inking, to *neat execution*.

CHAPTER II.

OPERATIONS IN EXECUTION.

SECTION A.

INSTRUMENTAL OPERATIONS.

Lines.

127. The only instrumental processes in *execution* are, the inking of single lines of various sizes, both straight and curved; and pen-shading, in ruled or compass-made lines, in imitation of the mild or bold shading, seen respectively in good steel and wood engravings.

128. Four kinds of lines are recognized in geometrical drawing, viz. *Full*, *Broken*, *Dotted*, and *Broken and Dotted* lines. A *full line*, whatever be its size, is one which is made by a continuous motion of the pen, from end to end of the line. Such lines are the top ones, *a, b, c*, of each group in Pl. III., Fig. 1. Likewise, a *broken line*, as *d, e, f*, is one which consists of a series of dashes, from about a quarter to a third of an inch in length, and with but small blank spaces between them. These dashes should be close together, since, if far apart, the line will have a ragged appearance; also they should be equal and equidistant.

A *dotted line* is strictly a succession of dots, as at *g, h, k*. When thus made, it is often done by hand with the "mapping" or other fine pen. With the drawing-pen it is easier to make a succession of *very short* dashes, as at *m, n, o*. We thus have the two varieties, a *dash dotted line*, and a *point dotted line*.

129. *Broken and dotted lines*, as their name implies, are a combination of the two last named kinds of lines, and for appearance sake, perhaps, are almost invariably used in practice in place of broken lines. They look best when the dashes are short and moderately heavy, and the dots very fine, with two or three of them after each dash. See *p, q, r*. In place of dots, short fine oblique marks of uniform length and inclination may be used, as at *p*.

130. Of each of these four *kinds* of lines, three *sizes* are in common use, viz., Fine, Medium, and Heavy. Mere description alone cannot, of course, define these. Specimens are given in Pl. III., Fig. 1. Commencing with the *fine* line as a standard, it may be said to be the finest line which can readily be made by a good operator provided with a good pen.

131. Both the *kinds* and *sizes* now described are repeated as circles in Pl. III., Fig. 2. To ink good dotted circles is a work of some care, in order to avoid wearing out the centre. It is well, therefore, to learn to dot them by hand with the "mapping-pen," or other like pen. Groups of straight lines, of various *sizes* and *kinds*, should also be drawn by the student through a point of intersection common to each group.

The illustrative figures given in a text-book are necessarily small. The student may profitably fill one plate, of the size given in this work, with parallel straight lines only, in three groups, with two lines of each kind in each group. A second, and very neat plate, may then be composed of groups of circular arcs, symmetrically arranged in the upper half of the plate, and rings, or other regular lines of small circles of regularly graded size, on the lower half. A third plate may then contain groups of lines, some of which shall *meet only* at a common point, while others shall *cross* at their intersection. All the lines of each group should, for good effect, be of the same kind and size. Practice should then be had in making irregular curves, both full and dotted, by hand.

Remarks upon the Inking of Lines.

132. (a.) To give a *bright* and *clear* effect to a line-drawing, the lines should be very black; and not rusty, as if inked with diluted ink. For this purpose *Japanese ink* is best, and least liable to wash out.

(b.) To give what may be called a *clean* or sharp and smooth effect, the lines should not be ragged. To this end, the pen-point should be in a good condition; should be steadily pressed against the paper and very lightly pressed against the ruler.

(c.) To give what may be called an *even* effect, each line should be of the same size throughout. This result is secured by keeping the pen parallel to itself, or in the same relative position to the paper and ruler, while the line is being drawn.

(d.) In inking drawings which are to be tinted and shaded, and where it is desirable not to have black ink lines appear, the lines may be inked with *pale ink*, so as to enable one to clean off all pencil-marks before shading; or, being inked black, they may be

made faint by partially washing them out. The first method is, of course, the most expeditious.

(e.) In making a complicated drawing it is better to *pencil, and then ink, portions of it alternately*, than to pencil the whole before inking at all; for thus confusion is avoided, reference to parts already finished is facilitated, and the drawing is kept clean and in a condition to receive good ink lines.

(f.) It is a very familiar fact among draftsmen, in their early practice, that a pen will mark smoothly and freely alone, but not when drawn along against the edge of a ruler. To obviate this difficulty, let the pen merely rest against the ruler without pressing it, as such pressure closes up the blades so that they will not shed the ink. Press the pen moderately upon the paper, letting it lightly rest against the ruler, and keep it perfectly parallel to itself while it is drawn rather slowly over the paper. The heavier the line, the more slowly the pen should be moved.

(g.) In inking fine lines, special care must be taken to keep the ink free in the pen-points. This can be done by occasionally stroking the pen on a bit of soft paper, rag, or wash-leather.

(h.) Very small circles (Pl. III., Fig. 3) may be inked with the bow-pen, held vertically, and turned rather rapidly and with a steady twirl. Each one must, however, determine the method which best suits himself and his instrument. Such circles may sometimes be made by hand, though when unskilfully or carelessly done, this procedure spoils an otherwise fine drawing.

(i.) To avoid a broken appearance at the junction of straight lines with circles, it is better to ink the circles first, especially if a circle is to be drawn tangent to two straight lines.

(j.) When many concentric circles are to be drawn, the transparent horn, or other instrumental-centre, should be used, to prevent wearing an unsightly hole at the common centre.

(k.) The convexity of the pen-blades prevents their point from applying closely to the edge of the ruler; hence, before inking a line, carefully adjust the ruler to a convenient distance from the line.

(l.) In inking a group of intersecting lines, each of them can be made to pass through the common point of meeting better if it is drawn outward from that point; but if the lines are not very fine, the ink of any one just made will be drawn out into a blot in inking a new line. Hence let each line get dry before inking a new one.

(m.) Practice should also be had in inking tapering lines, both

straight and curved. Lines are thus inked, partly by successive partial closings up of the blades of the pen, and partly by dexterity in the adjustment of the position of the pen as to the paper and the pressure upon it.

Pen-Tinting and Shading.

133. [To avoid the monotony of too long practice with one instrument, the teacher may at discretion practically classify the processes of this "DIVISION" according to their *superficial appearance*, as *uniform* or *shaded*, instead of according to the *instrument* employed, as *pen* or *brush*. Thus, practice in *tinting* may be introduced here from the next *Section*. The student can then return to *pen-tinting and shading*, and from that proceed to *brush-shading*, thus affording agreeable alternation and variety.]

It is sometimes desirable to distinguish the areas enclosed by certain lines of a drawing by filling them with fine and equidistant parallel full lines. Such lines are usually drawn to make some acute angle with the edges of the paper, and being very numerous, must be made equidistant by the eye alone.

The least irregularity in the size of the lines or in the spaces between them destroys the uniformity of the shade produced by their use. These irregularities are less noticeable when the lines are made not very close together. On all accounts, therefore, $\frac{1}{4}$ of an inch is a very good distance apart for such lines, and this distance should rarely, if ever, exceed $\frac{1}{32}$ of an inch, and never, unless in very small figures, need be less than $\frac{1}{32}$ or $\frac{1}{64}$ of an inch. When a surface is thus lined, it is usual to leave a small space blank near some of the edges. Accordingly, let the learner imitate Pl. III. Fig. 4, in his practice.

To afford complete practice in pen-tinting, let a plate be formed containing six figures in two rows; one triangle, one rectangle, and one circle in each row. Then let the figures in one row be completely filled with shade lines, while those of the other row shall have narrow blank spaces left half around their perimeters, or against one side in the triangle. The remainder of the plate may contain very small figures very closely lined, and one or more figures divided by a line, on each side of which the shade lines shall proceed in different directions, as in Pl. III. Fig. 4.

134. Where it is desired to produce a regular and *well-marked* gradation of shade over a surface by means of ruled or circular lines, these lines must regularly *diminish in size and become fur-*

ther apart, from the darkest to the lightest part of the surface. See Pl. III. Figs. 5 and 6.

Increase of distance between the lines, merely, without variation in their size, occasions only a faint contrast between the darkest and lightest part of the shading.

Such lines may be parallel, converging, or curved, according to the shape of the surface to be shaded.

135. *Converging shade lines*, as in Pl. III. Figs. 6 and 9, are somewhat difficult to manage, and may be made to taper as they approach the vertex. Or else, they may be made parallel to an outside line of the group.

Curved shade lines, as in Figs. 7 and 10, are easily drawn when the lightest spot of the shading has the common centre of all the shade lines for its centre, but become more difficult when the lightest spot is not at this centre; for then the shade lines must taper as they approach the light spot. Otherwise, in the latter case, the shading may be produced by uniform fine circular shade lines, together with *radial* shade lines of different sizes—heaviest and closest together where the shading is to be darkest.

Let the student first study carefully, and then imitate the groups Figs. 5, 6, 7, and 8, 9, 10, Pl. III., which give practice in the use of *parallel*, *converging*, and *curved* shade lines.

136. Drawings, and the wood, or lithograph line engravings which represent them, may be plain, half-shaded, or darkly shaded. Half-shading requires little or no change in the *size* of the lines used. Heavy shading would be executed by means of very heavy and black lines as close together as possible at the place of darkest shade, and thence grading their size and distance apart.

SECTION B.

FREE-HAND OPERATIONS.—§ 2. TINTING AND SHADING.

Flat Tints in Ink and Colors.

137. For tinting with Indian-ink or colors the following materials are necessary. The best of Whatman's thick and rough, or "cold-pressed" paper, wet on the face, or quickly dipped into a tank of clean water, to remove superfluous sizing, before being strained upon the drawing-board; a cup of clean water; several camel's hair brushes, which can easily be brought to a fine point, and one of which may conveniently be of large size; a clean sponge, and a tile or cup in which to prepare the ink.

138. In the operations of tinting the materials are simple, but everything depends on their proper use.

The following important points may be noticed :

(a.) Keep the paper, and everything which is to touch it, perfectly clean, and especially free from the slightest greasiness.

(b.) Having a hard and sharp pencil, pencil in very faint lines the outlines of the surface to be tinted. These lines are *not to be inked*, unless it be with *very pale* ink, which may be done, particularly if the outline of the figure is quite complex.

(c.) Avoid touching the India-rubber to that part of the paper which is to be tinted, either before or after the tinting is actually done, as the paper will thus become abraded so as not to receive a uniform tint, or the tint will be partially erased.

(d.) The drawing-board should be slightly inclined, so that the ink will flow freely after the brush, and not settle in spots.

(e.) The first and most common errors in tinting consist in applying the ink either *too dark* or *too dry*. If applied too dark, it will dry in streaks, owing to the presence of so much color in the water that it cannot distribute itself uniformly; and if applied too dry, it will dry in streaks also, since each stroke of the brush will in that case necessarily make its own distinct mark.

(f.) No absolute rules can be laid down relative to the best size of brush, or to the necessary amount of moisture to be contained in it. These vary with circumstances. In general, however, the larger the surface to be tinted the larger should be the brush, and the more full it should be, in order to yield an even tint over such large surface. Since a small brush soon gets dry, thereby giving a weaker tint, a large brush, provided it have a sharp point, is best for all but very small surfaces.

(g.) To give a rule as definite as possible : Let the brush contain a tint of moderate darkness, as seen in Pl. III. Figs. 11, 12, and let it be just wet enough to allow the whole surface to be kept wet till it is finished; for if an unfinished edge of the tint becomes dry, it will show through all the coats which may overlay it.

(h.) No after-tint should be laid on till the preceding one is perfectly dry, otherwise the two will commingle, forming but one potty tint, which will unduly soften the paper and leave ragged edges. Dark tints are made by a succession of light ones.

(i.) If the surface to be tinted be very large, it may be advisable to go over it first with a large brush moderately wet with clean water, to aid in keeping the tint from drying in spots. Also, it

such cases, as one large brushful may not be enough to afford a uniform tint to the whole surface, let a second or third brushful be dropped upon the surface before the first begins to fail. Or, use a broad flat brush. The drop that remains on completing the tint may be taken up with a clean damp brush.

(j.) Sometimes, however carefully a tint is done, some individual brush strokes will show. This, if not owing to defects in the paper, may be avoided by holding the brush vertically, and working it lightly.

Though a tinting brush should be kept scrupulously clean, yet particles of old ink or dust may be held in its roots. Hence, if a slow and heavy stroke be made, holding the brush flatwise, these particles will be pressed out and left on the paper just as chaff and chips are deposited on the margin of a stream. Therefore hold the brush vertically, and let a drop of pure tint escape to the paper; and then, keeping the brush as nearly vertical as possible, operate both rapidly and lightly to spread the tint throughout the surface and still to keep the edges perfectly even.

(k.) In tinting a large surface, especially, the drawing-board being conveniently inclined, keep the unfinished edge of the tint as straight from right to left as possible, and quite wet; and let the tint advance uniformly by alternate strokes from left to right and back again, keeping the brush vertical and the edges even.

These directions may seem tediously extended, but the securing of an even flat tint is often the most troublesome thing to the young draftsman—hence the minuteness of this description.

139. It would unnecessarily increase the expense of a work like this to have in it engravings of all the plates of a progressive course of exercises; hence the following list is offered, in working out which the student may profitably exercise himself.

1st. A plate of triangles, rectangles, circles, and small irregular figures, six or eight of them, with half of them bordered by heavy black lines, put on after the tint is dry, to give additional practice in smooth line-drawing.

2d. A plate of four rectangles, each about $1\frac{1}{2}$ ins. by 4 ins., with one tint on the first, two on the second, etc., to produce variety.

3d. A plate with two rectangles each, about 3 ins. by 5 ins.

4th. A similar plate with very wide black lines around the tint to give practice in ruling lines, with the pen opened wide. Observe, that in this case the ink should be quite thick; and, having the board inclined, rule "down hill" to avoid blotting.

5th. A plate with one rectangle, about 5 by 7½ inches.

6th. A plate containing a large circle and one or two moderately irregular figures.

7th. A plate with one large and very irregular figure, having interior blank spaces, so as to give a great extent of edge to be kept smooth, together with a large surface to be kept wet.

8th. A plate of six colored rectangles—one being tinted with prussian blue; one with burnt sienna; one with carmine; one with mixed carmine and burnt sienna; one with mixed blue, burnt sienna, and a trifle of ink or black; one of mixed blue, carmine, and black; all in pale tints, to afford exercise in judging of colors. The two last-named mixtures are obscure colors, which, with others, are called "neutral tints," and are sometimes used instead of ink for shading. The first of these neutral tints is greenish; the other is purplish.

SHADING IN INK AND COLORS.

1°. *Brush-Shading in Ink only.*

140. The bare *process* of shading is one study, and the distribution of shade over bodies of different forms is another study. We shall here consider exclusively the mere process of shading.

First.—Shading, considered merely as a process, will be in parallel tints, in converging tints, or in curved tints.

Secondly.—The process is divided, in general, according to the size of the surface to which it is applied, into the two operations of *preliminary shading* and the *evening up* process, or *final shading*.

Thirdly.—The process is further divided, as more particularly affected by the sizes of the surfaces, into *wet shading*, by *softened tints* and *coarse "stippling,"* as applied to large surfaces, anything larger than a rectangle of 2 ins. by 3 ins.; *moist shading* by *flat tints* and *fine "stippling,"* and *direct shading* with the brush nearly dry, a method which is applied to very small surfaces, and by which the shading of them is finished completely as it proceeds, by one operation, as if done with a lead pencil. By "stippling," meant the operation of filling up irregularities in the shading so that an even gradation of shade is obtained. The term is borrowed from the engravers, who use it to express that kind of engraving in which a surface is filled up with fine dots.

Shading may also be light or dark.

pure water where the softening off of a tint is to be effected. This is to be done *before* the tint is applied; then, when the tint reaches the watered band, it will spread through it, and very evenly and clearly shade itself off into the adjacent blank paper. This method requires care as to the relative wetness of the tint and the water band; and the size of the latter. These matters can be learned by practice on figures of various sizes.

144. *Remarks.*—(a.) The *relative* difference of shade, between the ends of a rectangle thus shaded, will depend on the number of divisions made in it.

(b.) This relative difference will evidently be diminished by every tint which covers the whole surface, after the first tint which extends over the whole rectangle.

(c.) The method according to which the *last* tint goes over the whole surface, rather than the first, is preferable; since, by the latter mode, each less extended tint would tend to wash up the preceding tint, where its edge was softened down.

(d.) Rectangles may be similarly shaded in colors; and a collection of beautiful plates may be made by any one having a large box of colors, by shading one rectangle with each color.

145. Pl. III., Figs. 13, 14, 15, and Pl. IV., Figs. 1, 2, illustrate *dry shading* and *brush stippling*. Pl. III., Fig. 13, and Pl. IV., Figs. 1 and 2, illustrate shading in *parallel* bands. Fig. 13 shows the appearance of the work after completing the preliminary process, and Pl. IV., Figs. 1 and 2, indicates, by line engraving, the appearance of the finished shading. In the preliminary process, begin at that edge of the figure which is to be darkest, and make a narrow stripe, as at the left side of Fig. 13. In doing this, the brush should be so nearly dry that the tint will dry about as fast as it touches the paper, or so that by the time the bottom of the stripe is reached the top will be dry, and ready to be overlaid with a new tint. The first tint having become dry, begin again at the left-hand edge, and make a little broader stripe, and so proceed over the whole figure, letting each stripe extend back to the left-hand edge, and also letting it become dry before a new one is added.

By having the brush but slightly wet, the tendency of the successive stripes to settle into black lines on their edges will be very much diminished. Several figures can be shaded at one time, since, as soon as one becomes so wet as to begin to swell the paper, others can be worked upon till this one dries.

146. This preliminary process being well worked out, the shading is finished by taking a little light ink in a damp sable brush

and by shading off the successive stripes as with a lead-pencil, until their edges are obliterated, as in Pl. IV., Fig. 2.

147. *Remarks.*—(a.) In conducting this finishing up process, the brush should be but slightly wet; *first*, to avoid washing up the previous tints; *second*, because the strokes, while wet, appear darker than when dry, and so deceive the eye; whereas, if the brush be so dry that each stroke dries as fast as it is put on, the effect can be seen at once, and time will be saved; *third*, so fine a point can be retained that the smallest light specks in the shading can be filled up without overlaying surrounding darker portions of shade; *fourth*, because with the brush in this condition it can be worked with as if it were a lead pencil, its strokes being light and fine, or broad and dark, according as it is touched lightly or vigorously to the paper.

(b.) Notwithstanding the last remark, the color of the ink in the brush should be somewhat proportioned to the darkness of the part of the shade to be evened up. Thus, in Pl. III., Fig. 13, lighter ink would be used in evening up the right-hand, than the left-hand portion of the figure.

(c.) In shading on rough or "cold-pressed" paper, it must be borne in mind that the action of the light on the prominences of the paper itself, is such as to make the shading appear uneven, unless viewed by a light coming in the same direction as that in which the shading was executed.

Hence, shading on rough paper should be begun and completed, and always viewed, by the same light. Shading on very smooth paper will look about equally well in all lights.

(d.) On account of its nearly uniform intensity in any given direction, a north light is preferable, and should be employed if possible.

(e.) The absolute darkness of the darkest part of the shading seems to be a matter of constitutional preference. Some involuntarily make this darkest portion black; others make it as light as the right-hand part of Pl. III., Fig. 13, and still others invariably adopt a medium darkness, leading towards the very dark, as seen in the figures.

Evenness, however, and not the absolute darkness of the shade, is the main thing to be sought.

(f.) To continue the shading till it fades away into white, commence with much lighter ink where the first few dark bands end, and go on as before, but without going over the previous parts any more. Two or three such changes of ink, with the after-pro-

cess of evening up, will soften down the shade to an imperceptible lightness.

(g.) The limit at which the method changes from that now described to that of softened tints, Pl. IV., Fig. 3, is arbitrary. Figures of sizes between Pl. IV., Figs. 1 and 2, may properly be shaded by the method now explained.

(h.) In shading on tinted papers a brilliant effect may be produced by tinting the lightest part of the shading with *white* paint, commencing at the part *designed to be lightest*, and going back by successive stripes towards the darkest part.

(i.) The foregoing explanations apply to shading in *converging bands*, Pl. III., Fig. 14, except that the bands are then triangular; also, to shading in *curved bands* around a lightest point, as in Fig. 15, instead of towards a lightest line, as in Figs. 13 and 14.

148. Finally, *the direct process* of shading a surface with a damp brush, used as a lead pencil would be, needs now no particular description. It is applied to surfaces so small that the preliminary process of successive tints is unnecessary.

When a surface is long, but very narrow, it is shaded either as just described, or by ruling one or more tint lines with the drawing-pen, not as in Pl. III. Fig. 5., but touching each other.

2°.—Of Brush-Shading in Colors.

149. Other colors than ink are sometimes used in shading. There are two methods of shading in colors, aside from the modes as affected by the *size* and *shape* of the shaded surface. The *first* method consists in shading the surface at first with ink, as already described, and then in tinting it over with one or more flat tints of the desired color. The *second* method is to use the color only, operating with it just as has been described in treating of clear ink-shading. A warm reddish-brown called "*sepia*," a cold dark-brown called "*bistre*," the various neutral tints formed by mixing browns and blues, and the grays formed by mixing black and white, or deep crimson with pale blue and yellow, are used according to the second method, while the first method is often adopted when clear light colors alone are to be used.

150. Light is white. Degrees of darkness are degrees of absence of light, *i.e.* degrees of blackness. Accordingly it seems preferable to shade in ink only, and to tint over it in flat tints of color; for the *absolute tint* pertaining to the substance of the thing drawn is *uniform*, while its shade may be graduated accord-

ing to its fulness of exposure to the light. In shading with colors, on the other hand, both the shade and the tint are made variable.

3°.—*Of free hand Pen-Shading.*

151. This kind of shading is of two kinds. The *first* consists of two or more sets of lines crossing each other, and varying in number and fineness. The *second* is pen stippling, consisting of innumerable fine and equal dots, distributed so as to produce an even gradation of shade, as seen in the faces, and other finer parts of bank-note engraving. The latter method, when well done, gives a very fine effect, and is hardly, if at all, more laborious than equally good dry brush-shading.

§ b.—OF THE CONVENTIONAL TREATMENT OF MATERIALS USED IN CONSTRUCTION.

Graining of Wood.

152. Wood is imitated exactly enough, in geometrical drawing, to enable an observer to know that wood is meant to be represented, but not so exactly as in the imitations of particular kinds of wood made by the best decorative painters.

In geometrical drawing, distinctions are, therefore, usually made only between the graining of "soft" woods of all kinds, and the hard woods as a class; between the graining of wood, as seen in line drawings, and as seen in shaded and tinted drawings; between graining of timbers drawn on a large and on a small scale; and between "heart-wood" and outside pieces. These cases are illustrated by the figures of Pl. IV, now to be described: in making which some fine pen (64) is used.

Fig. 4 represents a timber as it would be grained in a line drawing. The ink used should be pale, and the lines fine. The groups of marks which surround the knots are made first; and of those marks which make up a group, the outer one is made first, since the shape of the group can thus be more readily controlled than by making the innermost one first. As in the figure, these marks should be as close together as possible in the direction of the width of the timber, but with a considerable space left between their ends. Having made the knots, each group of marks being, as a general rule, at least as far from the next as itself is long, the rest of the timber is to be filled up with fine and slightly wavy lines, which shall run through its whole length, without touching each other, or the groups around the knots.

153. The right-hand end of the timber is represented as broken. This breakage is to be represented by large irregular points, broken up into finer ones, as shown in the figure.

It will generally be found most convenient to turn the board so that the pen can be drawn towards the body, and not from left to right, in making the graining.

154. If the last figure represent hewn timber, Fig. 5 may be considered as representing sawed timber, and the graining line are splintered or discontinuous.

The shading of the knots, which is, in the succeeding figures, made with a brush, is here made with a pen, and, by a sort of dotting motion, the lines outside of the knot-marks are made to have a splintered appearance. Figs. 4 and 5 may be tinted with the usual color given to wood in tinted drawings, viz. a very pale tint of burnt sienna, resembling newly planed pine wood.

155. In Fig. 4 the shading of the knots, and of the sides of the wide spaces between the knot-marks, is supposed to be made with a fine-pointed brush, taken nearly dry, which is the usual method.

Variety in the knots, or looped sections of the grain, may be made in imitation of newly planed boards having a strongly marked grain. In case a drawing were shaded, but not tinted, the tint might be omitted from the wood-work.

156. For tinting "hard pine," a little carmine may be added to the burnt sienna. Fig. 6 represents hard wood. The little transverse marks which distinguish it from the preceding figures, are made after the knots and before the longitudinal marks, which are interrupted by the transverse marks. Burnt sienna, softened with a little white or yellow ochre, are better tints for oak than is burnt sienna alone.

157. When a timber is represented as very narrow, the groups of knot-marks should take up its whole width; but if a large timber be represented on a moderately large scale, so as to be about half an inch wide on the drawing, let the knots be made no larger than in Figs. 4 and 6 of Pl. IV.

158. The timbers thus far shown have the appearance of heartwood, or of boards whose widths are whole chords of a log. On a board, which is but a small outside portion of the widest section of the log, the knot-marks would be mostly omitted and the longitudinal marks only would be made.

159. The graining of ends of timbers is shown in Pl. IV. Fig. 5; α is the end of a square timber, and is grained in concentric rings, representing the growth of the tree; and ragged radial

marks, diminishing outwards, which represent cracks. The rings of a moderate-sized end, as *b*, may have an imaginary centre somewhere outside of the timber; and the end of a plank, *c*, may be grained in straight lines.

160. If a timber be cut lengthways, and if it be desirable to distinguish the cut surface from the body of the timber, it may be done in a line-drawing, by drawing fine parallel lines, as in Pl. III. Fig. 4; and in a tinted drawing, by omitting the graining and retaining only the tint, as in the right-hand part of Pl. IV. Fig. 6, or by substituting parallel longitudinal lines for the ordinary graining.

161. In representing timbers on a very large scale, as shown in Pl. IV. Fig. 7, which is supposed to form part of a tinted drawing, the graining marks and shading may be made by a brush with tolerably dark burnt sienna, mixed with a little ink, and then the whole may be tinted with clear light burnt sienna. A timber may be supposed to form part of a drawing which is shaded with some of the grays or neutral tints already described. Here the natural grain, which may be considered as somewhat effaced by weather staining, is executed wholly with a *moist* brush, and the timber is then tinted over with some neutral tint.

162. The manner of graining of the ends of these large timbers is sufficiently evident from an inspection of Pl. IV. Fig. 7, *b*.

The foregoing may serve as a sufficient notice of the treatment of the common woods. Whatever variations from the patterns of Pl. IV. are made by the student, let them, therefore, result from observation, and not from mere uninstructed fancy.

163. Geometrical draftsmen may sometimes be called on to make ornamental drawings, as of locomotives, fire engines, or other decorated machines, in making which it may become desirable to imitate the valuable woods—mahogany, rosewood, or black walnut. These woods are represented in general by laying on the colors, not in tints, but in thicker coats, by which process the separate strokes of the brush, as they overlap each other, will of themselves represent the variegated tints of such woods.

164. For the color of mahogany, burnt sienna, mixed with a portion of "dragon's blood"—a dark, dull red—makes a good imitation; for rosewood, the same, with some carmine; and for black walnut, burnt umber, with a minute quantity of dragon's blood and white, will answer the purpose.

Masonry.

165. In the figures illustrative of the mere *processes* of represent-

ing masonry, only simple piles of regular blocks have been represented, nothing more being necessary. The representation of masonry may be considered under the following heads:

Brick.	{	Common,
		and
		Fire-brick.
Stone-Work.	{	Ashlar.
		{ Plane.
		{ Rough.
		Rusticated.
		Rubble.

Also, both brick and stone-work may be made in line drawing or in tinted drawing.

166. Line drawings, both of brick and *plane ashlar*, or smooth and square-faced stone, are made as shown in Pl. IV. Fig. 15, *i. e.* merely by inking their outlines. The section or cut surface, Pl. IV. Fig. 8, may be distinguished by fine wavy lines, leaving a small blank space for distinctness' sake at the upper and left-hand edges of each stone.

167. *Rough Ashlar*, that is, squared stone-work, in which the *face* of each stone—or the part seen in looking at the wall—is left rough, is shown in line drawing at Pl. IV. Figs. 9 and 16, by an application of pen-shading, chiefly to the right-hand and lower edges, to give distinctness and uniformity to the whole. The treatment of the different stones in the figure indicates various degrees of roughness. By making several sets of shade lines, of variable length in each set, as in Pl. IV. Fig. 16, a harsh edge to the shading may be avoided.

168. *Rusticated Ashlar* is that which is cut off or "*chamfered*" at the edges. It is shown in line drawing by a rectangle just within the outlines of the stone, and of which the lower and right-hand sides are heavy—Pl. IV. Fig. 12. These latter sides alone are sufficient in drawings made on a small scale. See Pl. IV. Fig. 17.

Stone which is wrought on the face with a pick or pointed hammer, is said to be *nigged*, and is shown in Pl. IV. Fig. 17.

169. *Representation of Masonry in Tinted Drawings.*—Common brick-work is, in tinted drawings, represented by a tint of carmine and burnt sienna, with the addition of some ink, over which, lines, as in Pl. IV. Fig. 8, are made when a section of the work is to be shown. An extremely light tint of burnt sienna and ink, or better, of gamboge and a little ink, will answer for fire-brick.

170. Building stones are very generally of some bluish gray color. They are, therefore, tinted with a light color composed of black and white (pure gray) to which is added Prussian blue, with

a little carmine. By varying the proportions of these colors, or by changing the kind of blue, or omitting the carmine, different stone colors can be made. To avoid the too smooth appearance which the tint alone would give, the stones are filled up, as seen in Pl. IV. Fig. 10, with fine vertical marks or "hatchings." Concerning these hatchings, observe—

- (a.) That they should be made in pale ink.
- (b.) That they should be in their *general direction*, vertical and parallel, but that—
- (c.) They should be slightly wavy, and not rigidly straight as if ruled.
- (d.) They should not be like accents (') heavy at the top, but of uniform size; hence each must be made deliberately.
- (e.) They should be arranged in curved rows, varying or different stones, to avoid a monotonous appearance, and sharply curved to give a decided effect.
- (f.) For distinctness and to represent the mortar, narrow blank spaces should be left at the top and left-hand edges.

171. *Rough Ashlar* is represented in tinted drawings either by the pen or brush, as seen at Pl. IV. Figs. 9 and 11; and *rusticated ashlar* is shown by tinting the space between the double outlines with ink, on the lower and right-hand sides, before the stone tint is put on. See Pl. IV. Fig. 14.

172. *Rubble* consists of rounded or angular irregular stones, whole or broken, or of flat irregular slabs, as of slate.

A fragment of rounded rubble wall is shown in Pl. IV. Fig. 13.

173. *Brown freestone* may be well represented by a dark tint composed of burnt sienna, carmine, and ink. If rough stone be intended, this tint may be purposely laid on unevenly—darker in spots—on each stone before the "hatchings" are added.

174. The broken edge of stone-work, Pl. IV. Fig. 8, should be represented by a *smooth* and yet irregular line, not by a finely-splintered line, as in the case of wood-breaking. Pl. IV. Fig. 18, represents a stone ornamented with variously curved cuttings. Such stone-work is called *vermiculated*.

Rock, Earth, and Water.

175. Pl. V. Fig. 5, shows a section of a stream whose bed and banks are of *earth*, and as it would be made in a line-drawing. The water is represented by fine horizontal lines, which at the top are very close together and gradually become further apart. The

banks and bed of the stream are made by a sort of roughly-shaded pen-work, darkest at the surface line.

176. Pl. V. Fig. 6, shows a section of a stream having the sides and bed of "stratified" rock—that is, rock naturally arranged in layers.

177. Proceeding with the description of the figures of this plate, Fig. 1 represents earth again, in a little less bold style of execution than is shown in Fig. 5; Fig. 2 represents sand; Fig. 3, gravel, with scattering stones; and Fig. 4, rough and broken rock, such as is not found in regular layers.

178. We will next describe the manner of representing earth, etc., on colored drawings. In Fig. 5, if colored, the water would be shown in blue lines, and the earth by strokes of mixed burnt sienna and ink, mingled with darker strokes, having more ink and a little blue added to the burnt sienna. These dark strokes prevail in the bed of the stream, in order to indicate mud, while the banks take on a lighter and more gravelly appearance towards the top.

Notice, in all the representations of earth, that the surface-line is full and distinct, the thickness of dirt represented is small, and that the pen or brush-strokes shade off gradually into the blank paper. Also, all the brush-strokes—laid on nearly dry—should be horizontal, and *not* parallel to the surface-line of the soil.

In colored drawings, sand would be represented by *pale* dots on a tinted ground, roughly shaded from the surface-line, after the method of "wet-shading" (142). Gravel is similarly shaded and coarser. In representing rough rock, the tint is darker in spots to aid the shade-lines running in various directions in representing the irregularities of the surface.

Metals.

179. *Iron* is the chief metal to be represented in geometrical drawing. Occasionally brass, copper, and silver are to be represented. Two tints are used for iron; the first is a light tint of clear Prussian blue, which is the usual conventional tint for wrought-iron; the second, for cast-iron, is a gray tint composed of black and white with a little indigo. The Prussian blue may properly enough be used for very small or for ink-shaded surfaces; but for very broad unshaded surfaces, it is rather too vivid, and a more subdued tint, like the mixed one just named, or one made of blue, with a very little ink, is preferable.

180. *Brass* is of two sorts, hard and soft; otherwise, dark and light. The latter is tinted with clear gamboge, or with

"King's yellow;" the former with the same color, to which has been added a very little vermilion or carmine. The addition of yet more of vermilion or of carmine to the clear yellow gives an appropriate tint for *copper*.

181. *Silver* may be represented by an almost invisible tint of blue; or, on a dark-colored ground, by white mixed with a minute quantity of blue, and laid on thick, and not in weak tints.

All drawings, in which it is desirable to represent white or very light-colored substances, are best made on tinted paper.

182. *The broken edge of metal* is represented by a line whose general direction is straight, but which is broken up by fine notches throughout its whole extent. See Pl. V. Fig. 16.

Slender rods of metal are often represented by single or double lines, according to the scale of the drawing, of the proper conventional tint—ruled with the drawing-pen—rather than by black lines with a tint between them.

183. *The learner is here cautioned* not to rely too much on the plates, or on any engraved plates, as models to be copied in the representation of shading, and materials used in construction. He should rather follow, carefully, the quite minute directions given in the foregoing text, and, for *models*, take fresh clean specimens of the materials themselves; newly planed woods, fresh chips of various kinds of stone, and freshly fractured bits of the ordinary metals. Next to these, as copies, the best accessible works of good professional draftsmen are the best.

Printed copies that are fine, are too costly for conveniently cheap text-books; but in connection with the foregoing, though subordinate to them as copies, it will be profitable to consult any of the following, or other like works, that are accessible:

The Engineer and Machinist's Drawing Book.

CASTELNAU: *Cours de Mathématiques appliquées.*

ARMENGAUD ET AMOUREUX: *Études complètes d'Ombres et de lavis appliquées.*

DELAISTRE: *Cours complet de Dessin Linéaire.*

WEISSENBORN: *American Engineering*, illustrated.

WEISSENBORN: *Locomotive Engineering*, illustrated.

WORTHEN: *Encyclopædia of Drawing.*

Colored French Lithographs, by various authors, some with explanatory text, and of various subjects.

DIVISION III

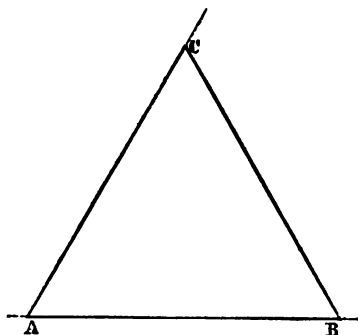
PRACTICAL CONSTRUCTIONS OF TWO DIMENSIONS.

CHAPTER I.

THE CONSTRUCTION AND COPYING OF SIMPLE FIGURES.

184. *To construct an equilateral triangle upon a given base.* Fig. 16. With the T rule draw the indefinite line, AB, and upon

FIG. 16.



it lay off AB equal to the given base ; then, since the angles of an equilateral triangle are each 60° , draw AC and BC with the 60° triangle, which will complete the figure.

185. *To construct a square upon a given base.* Fig. 17. Draw, as before, an indefinite line with the T rule, and on it lay off AB, equal to the given base. At A or B draw, with the triangle, an indefinite perpendicular, as BC. Recollecting that the diagonal of a square makes an angle of 45° with its edges, place the hypotenuse of the 45° triangle at A, and without drawing the diagonal, mark its intersection, C, with the perpendicular just drawn.

Through the point C, thus found, draw the horizontal line, CD,

with the T rule, and the perpendicular, AD, with the triangle, and the square will then be completed.

EXERCISES.—1. Draw an isosceles right-angled triangle on a line as AB, Fig. 16, drawn with the T rule, for its hypotenuse. Do this with the simplest use of the 45° triangle.

2. Make a line as AB, Fig. 16, one of the sides of the right angle of an isosceles right-angled triangle.

3. Draw a triangle of 30° , 60° , 90° , drawing its hypotenuse with the T rule.

4. Do. drawing its longer base with the T rule.

5. Do. drawing its shorter base with the T rule.

6. Draw an equilateral triangle on a line parallel to AD, Fig. 17, as a base.

7. On a line as AD, Fig. 17, as a hypotenuse, draw a triangle of 45° , 45° , 90° .

8. Make the line just described the hypotenuse of a triangle of 30° , 60° , 90° .

186. To draw a 60° rhombus on a given shorter diagonal. Fig.

18. Draw short fine dashes with the T rule where A and B will

FIG. 17.

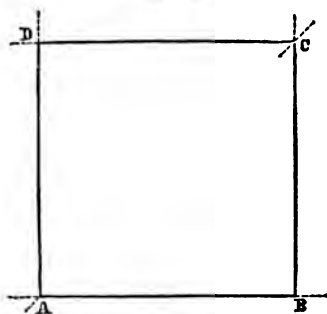
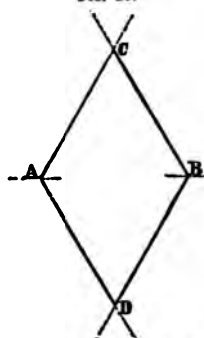


FIG. 18.

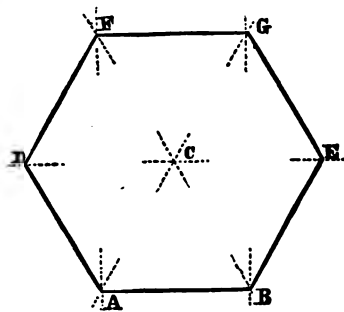


fall when the short diagonal is laid off. Then at A and B draw lines, as in Fig. 16; also similar lines, AD and BD, below AB. These lines will complete the figure.

187. To construct a hexagon upon a given side without the use of compasses. Let AB, Fig. 19, be the given side. Since the interior angles of a hexagon = 120° , the exterior ones = 60° , hence draw AD and BE with the 60° triangle. To find the length of these sides, consider that lines through A and B respectively parallel to BE and AD, will intersect at the centre of the figure; hence determine this centre, C, and through it draw a line, ECD, with the T rule, which will determine the points E and D. Then draw

DF and EG with the 60° triangle, and find where they intersect fragments of AC and BC produced, or fragments of perpendiculars

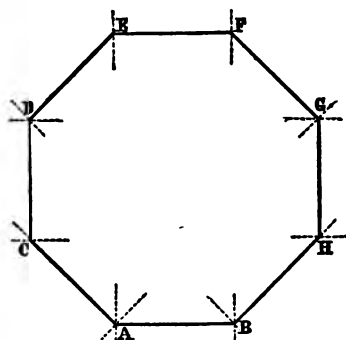
FIG. 19.



through A and B. Having thus found F and G, join them by the line FG, which should be parallel to AB, and which will complete the figure.

188. To construct an octagon with the least possible use of the compasses.—*First method.* Fig. 20. Draw AB, and on AC, drawn

FIG. 20.

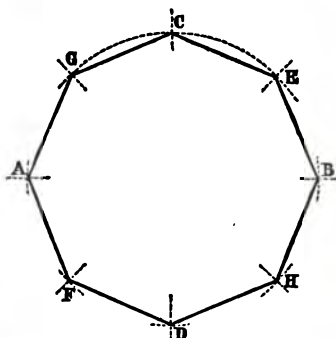


with the 45° triangle, lay off AC equal to AB. At C, draw the vertical line CD, and see where a line parallel to AC, through B, would intersect it. Having thus found D, E may be found by drawing DE with the 45° triangle till it intersects the perpendicular AE. Drawing BH, H may be determined by a horizontal line through C, or by a line parallel to AC, through E. G may then be located on the perpendicular HG, by a line parallel to BH,

through A, or by a horizontal line, DG. Finally, F is at the intersection of GF, parallel to AC, with EF parallel to AB; or at the intersection of GF or EF with CF, parallel to BH; or at the intersection of GF or EF with the perpendicular BF.

Second method. Fig. 21. Describe a circle. Without drawing

FIG. 21.



the diameters which are diagonals of an octagon, mark A and B, the extremities of the horizontal diameter; C and D, the extremities of the vertical diameter; and E, F, G, and H the extremities of diameters drawn with the 45° triangle. Connect these points, and the octagon will be complete.

Remark.—The foregoing figures may be drawn to scale from assumed dimensions. Thus the square, Fig. 17, having its side $AB = 1\frac{3}{8}$ inches, may represent a square whose side is 11 feet, drawn to a scale of 8 feet to the inch.

Stars.

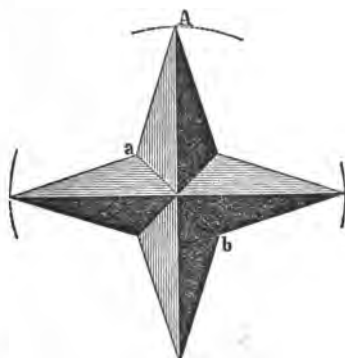
189. Stars, properly constructed, are in themselves beautiful, and are useful as ornaments in some kinds of drawings.

To construct a four-pointed star. Fig. 22. Let the points, as A, be at the extremities of horizontal and vertical diameters of a circle, and let the radius of the circle through the inner points, as a , be about one-third of the radius through A; the points, as a , being in the diagonal diameters of the inner circle—i.e. in the diameters drawn with the 45° triangle.

Join the points, as a , with the outer points, and with each other by diameters, as ab . These lines, with the diameters of the outer circle, form the outlines of the star, which may be shaded as in the figure. To shade the star evenly, ink its outlines with pale ink,

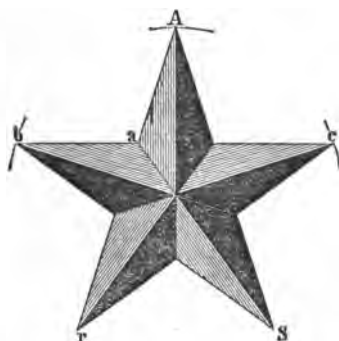
then tint the whole interior, and then with thick ink blacken the four sides which are made black in the figure, leaving a fine line of pale tint to separate the adjacent black sides.

FIG. 22.



190. To draw a five-pointed star. Fig. 23. Let the five equi-

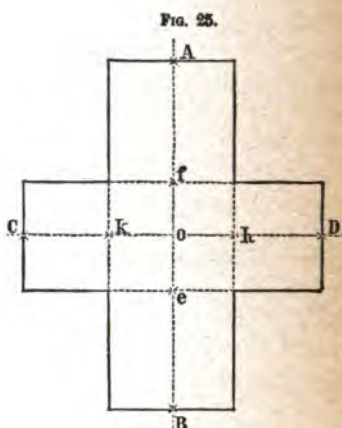
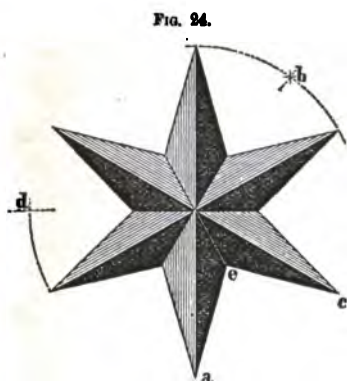
FIG. 23.



distant points, as A, b, etc., be in the circumference of a circle. A star of this form looks best when its points, as b and c, or A and r, are joined by straight lines; hence find a, by drawing bc and Ar, or bc and sa. Similar points will be in a circle through a. The completion and shading of this star can now be understood from the figure.

191. Fig. 24 shows a shaded six-pointed star, in reference to which it is now only necessary to say that its lines, as ae and ce, run to points b and d, 30° from the points opposite to a and c.

192. An *axis of symmetry* of any plane figure is a line which divides that figure into parts, which, if folded together about that line as an axis, will coincide. Thus AB, and CD, Fig. 25, are axes of symmetry, also called *centre lines*; since the half of the figure to the right of AB, for example, could be folded over upon the left hand half so as to coincide with it.



EXERCISE.—Draw an eight-pointed star, so that the internal angles, as *a*, Figs. 22, 23, shall be of 90° .

193. An isosceles triangle has *one* centre-line; a rectangle and a rhombus, each have *two*; an equilateral triangle, *three*; a square, *four*; a regular pentagon, *five*; a regular hexagon, *six*; and a circle, which has an infinite number of sides, has an infinite number of centre-lines. Whence we conclude that any *regular* polygon has as many centre-lines as it has sides.

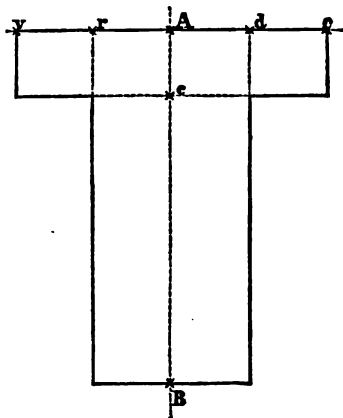
Fig. 25 evidently has two centre-lines, or axes of symmetry; Fig. 26 has one such line, and Fig. 27 has none. To *construct* Fig. 25, or any figure having two centre-lines, draw those centre-lines as AB and CD. From their intersection, O, lay off, on AB, O*f*

and Oe , each equal to half the width of the horizontal arm, and OA and OB , each equal to half the length of the long arm of the figure, and through these four points draw indefinite horizontal lines with the T rule. Then, on CD , lay off, on each side of O , Oh , and Ok , each equal to half the width of the long arm, and OC and OD , each equal to half the length of the short arm. Through these four points draw lines parallel to AB , and they will intersect the first group so as to complete the figure.

194. *To copy such a figure as the preceding by instruments.* Draw two new centre-lines through the point chosen as the representative of the point O on the paper which is to contain the copy, and then transfer to these centre-lines the same distances which were used in constructing the figure in the first place—*i.e.* if the copy is to be of the same size as the original. Call the new intersection of the centre-lines O' , and the axes $A'B'$ and $C'D'$, then if the copy is to be n times as large as the original, take $O'f$, for instance, and lay it off n times on $O'A'$ to determine the corresponding point, f' , of the copy; or if the copy is to be $\frac{1}{n}$ as large as the original, take one-*n*th part of each distance, as $O'f$, and lay it off as on $O'A'$, which will give the point as f' on the reduced copy.

195. If the measurements from which the original was constructed are accessible, and if the numerator of the fraction expressing the ratio of the copy to the original be anything else than

FIG. 26.



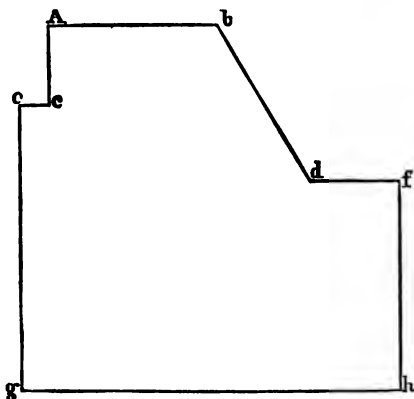
unity,—thus, if the copy be $\frac{2}{3}$ of the original—it will be more convenient to determine the scale of the copy and to construct it by

scale directly from the measurements. Thus, if O_f , which we will suppose to be an inch, represents 4ft. on the object, and if the copy is to be $\frac{2}{3}$ of the original, 4ft. will be represented by $\frac{2}{3}$ of an inch, 2ft. by $\frac{1}{3}$ of an inch, and 10ft. by 1 inch, so the copy may be constructed from the original measurements, on a scale of 10ft. to the inch.

196. Fig. 26 represents a surface having but one axis of symmetry, AB. On AB lay off the length of the figure; from A lay off $Av = Ae$, $Ad = Ar$, and Ac . Horizontal and vertical lines through these points will complete the figure. The methods of copying such a figure are evident from the explanations given in connection with the last figure.

197. Fig. 27 represents a surface having no axis of symmetry.

FIG. 27.



To construct or to copy it, begin at any point, as A, and lay off the measurements from point to point around the figure, or better, perhaps, lay off first Ab , then Ac , then bd , then ce , and see if h , as located through b , d , and f , coincides with h , as located through c , e , and g .

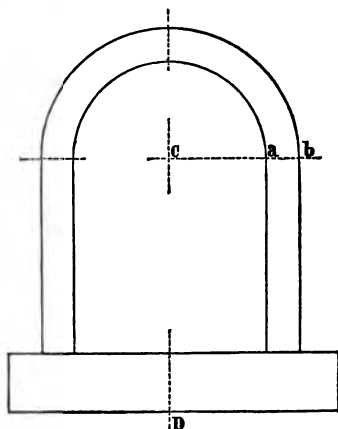
198. Fig. 28 represents a practical construction, having one axis of symmetry: viz. a semi-circular topped window. The student may draw this large enough to fill half a plate, and may shade the interior or window opening, as described in Arts. 142, 143.

Observe, in this and in all similar cases, that it is better to ink the semi-circles before inking the vertical sides of the window, since straight lines can be more neatly made tangent to a given arc than the arc can to two given straight lines. If shaded, this figure should be inked in pale lines.

Taking and Recording of Measurements. Construction of Plans of Buildings.

199. Among the number of useful practical constructions involving but two dimensions, plans of buildings are as important as any. In Pl. V. is seen the plan of a building, finished in a very complete

FIG. 28.



style of execution for such drawings. As this figure is supposed to be drawn from measurements, the manner of taking and recording those measurements is here given.

They are *taken* by measuring the length of the different parts of the walls, the width of the doors and windows, etc., with a foot-rule, yard-stick, or tape measure. These measurements, to be true, should be horizontal, and hence may be taken on the ground, or along a joint, in the case of the walls or on the floor for the doors; or on the sills of the windows. Also the measurements on any line, as cd , should not be taken from point to point, as from c to the window, then across the window, then between the windows, etc., but they should all be referred to the end of the line, as from c to the left side of the window, from c to the right side of the window, from c to the left side of the next window, etc.

In *recording* the measurements, one accent (') denotes feet, and two accents (") inches. The points between which the measurements are taken are indicated by arrow-heads; thus, on the figure, from c to the window = $4\frac{1}{2}'$ (feet), from c across the window = $8'$, and $cd = 21'$, etc.

200. The *plan of a school-house*, shown on Plate V., has, as a whole, no axis of symmetry, though its separate parts have such axes. Thus, a line through the middle point of ab , and perpendicular to ab , will be a centre-line for nearly all parts of the large wing of the building. So also a line perpendicular to cd , at its middle point, will be a centre-line for most of the parts of the smaller wing. The porch and adjacent windows of this wing are symmetrical with reference to a line perpendicular to the middle point of ef .

From these descriptions—and having the scale $\frac{1}{8}$ —the construction can be finished.

201. In regard to *execution*: first tint the entire plan with one dark tint, or two light tints of burnt sienna; next rule the boundaries of the blackened parts of the walls with heavy black lines, and across each window-opening ink three lines, besides the wall lines, to represent the window-sashes, observing the order of light and heavy lines as shown in the plate; then fill up the solid parts of the wall with thick, black color, applied with a brush; and lastly, rule the diagonal medium-sized broken lines as shown in the figure.

202. The full working-up of a design for a dwelling, belongs to the architect. But the plans of the several floors, showing the general arrangement of the apartments, can be made by every one. In fact, just so far as a house expresses the owner's ideas as to what it should be in respect to internal arrangement, he must be the designer of its plan.

The following analysis of the design in Fig. 29 may invite to the careful preparation of others as an exercise or pastime. Such exercises will tend to cultivate fine perceptions of taste and comfort in the allotment of apartments to the various elements of household life, and will induce skill in contriving combinations, rich in features of united beauty and convenience.

This design is adapted to ground west of a road which runs north, the ground soon descending from the road line. The principal story only is fully shown. The basement and second story are sufficiently represented, the former, when necessary, by broken lines, the latter by dotted lines.

A is the vestibule $9' \times 9'$ of a three-story tower, and is entered by triple arches on two sides.

BB'B" is a hall $7' \times 17'-6"$, besides the wing B', which is $8' \times 10'$ and contains the main stairs.

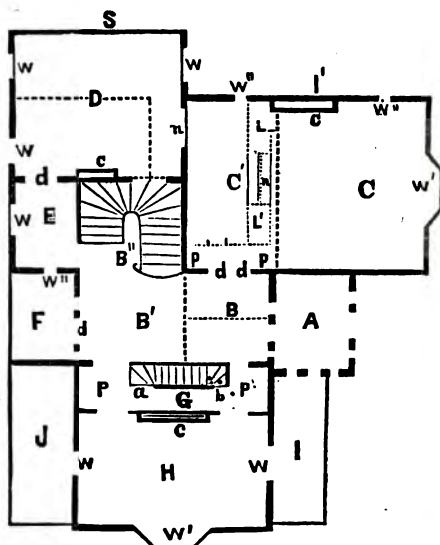
CC' is the parlor, $17' \times 25'-6"$, entered by folding-doors, dd , and provided with a bay-window, w' , and two narrow and high stained

windows, $w''w''$, the northern aspect being supposed to be unpleasant. D is the library or study, $14' \times 16'$, entered under the stairs, and provided with a shallow niche at n , and chimney, c .

E is a study, working, and apparatus, etc., room, $6' \times 10'$, with a narrow, stained window, w'' , and opening both into the library and main hall.

F is a porch opening into the hall.

FIG. 29.



G is the side hall including stairs to the basement, descending at b , and to the chambers, ascending from a , also passages, pp' .

H is a sitting-room, $12' \times 16'$, flanked by the broad piazza, I, which is entered through a long window. The south bay window, w' , admits light in healthful abundance, and affords a place for house-plants.

Passing to the basement—under C is the cellar. Under C' is a fuel and furnace room. Under B'B'' is the kitchen, lighted by a road window under the hall door, d , and a window in the partition of the wash-room, which is under E. Under F is the kitchen orch. Under B are the kitchen and dining-room closets. Under I is the dining-room, flanked by the greenhouse or grapery, J, which fronts the south and west. This room also has a south bay window under w' , and an underground wall on the east side. It

is therefore snugly sheltered. Under D is a bedroom and basement passages.

Proceeding now to the second story—over C is a chamber $14\frac{1}{2}' \times 17'$, with a niche, *n*, and closet, L. Over C' is a bedroom 9×13 , with niche and closet. Over B' is the hall landing, reaching over B'' far enough to afford a passage, *pp*, to the chambers C and C', and an entrance to the dressing-room over E. Over D and F are chambers. Over A and B is a second story sitting or working room, with stained bay window in the tower front. Over *p'* is a closet for the chamber H. Over the back stairs ascending from *a* are attic stairs, leading also to the third story of the tower.

203. *Modifications.* By placing the west window of H near *p*, and the chimney, *c*, adjacent to J, we gain a place for an organ niche at G, between *a* and *b*, and favor the warming of the greenhouse. Omitting the side hall, G, the stairs to the basement may be in B'', and H will be $16' \times 17'$.

By placing the library chimney at *n*, in the parlor partition, we gain space for a book-case niche under the turn of the stairs; and can then make under C' a wash-room, under E a bedroom, and under D a kitchen.

204. *Further modifications, for level ground.* Place a kitchen and wash-room on a level with H, over J, and put the side hall, G between them, and at right angles to its present position. Then make H the dining-room, and D the sitting-room, with an organ niche at *c*, and a large square bay window at S for flowers, and a piazza at I'. In this arrangement we suppose that the house faces the west instead of the east, as now. Finally, over A and B will then be the study, and E a family library, giving a very agreeable design.

Earth and Water Sections.

205. Pl. IV., Fig. 20, represents the section of a canal bank with the dimensions recorded, on a scale of eight feet to an inch; affording an example of the line drawing of the embankment of an artificial water-course.

The body of the embankment is of gravel; the dark portion between it and the stone facing, and above it, represents a mixture of sand and clay, called *puddle*, and forming the *lining* of the embankment to keep it water-tight.

Where the embankment is also used as a landing, as in cities, its stone facing sloping at 45° , is replaced by a vertical wall 8 feet high, $2\frac{1}{2}$ feet thick at top, and $3':2''$ thick at bottom, resting on a

plank platform supported by timbers parallel to the canal. The wall is vertical on its back, is lined with a vertical puddle wall one foot thick, and is edged at top with a timber a foot square, back of which is the top puddling. From this description the student can construct the section now described.

The embankment shown in the figure is the "tow-path." The opposite bank is a little smaller, and is called the *Berme*.

Traced Copying.

206. To make a *line-copy* for *temporary preservation*: fasten a piece of tracing-paper over the drawing to be copied, and, without the use of the common operations of construction, ink over the lines, as seen through the tracing-paper, with a drawing-pen.

To make a *portable line-copy* for *permanent preservation*. Proceed as before, only substituting *tracing-cloth* for tracing-paper.

207. To transfer a copy to new drawing-paper so as to make a new finished drawing. Make the copy on tracing-paper, as above described, then dust over the back with crayon or soft pencil scrapings; then lay the blackened side of the tracing-paper upon the new sheet of drawing-paper, and press over the lines of the copy with any smooth hard point. On removing the tracing-paper, after thus going over its lines, they will be found to be transferred to the new sheet. If the lines of the tracing-paper can be accurately followed on the back, it will be sufficient to go over all the *lines* of that copy, on the *back* of the copy, making broad lines with a soft pencil, instead of dusting over the whole back of the copy.

It is convenient thus to copy a drawing when a number of copies of the same thing, made to the same scale, and similarly finished, are desired; also when a drawing has been correctly constructed, but spoiled in execution, and when it is desired to take copies of letters or drawings, of which the originals cannot be obtained.

208. *Another method of copying*, called "*pricking off*," is, to tack the original drawing over the new sheet, and then, with a fine needle-point, to prick through into the new paper all the points of the drawing; but this plan requires especial care; for, *first*, if the original drawing be good, it is undesirable to puncture it; *second*, if all the points be thus pricked off, it will be difficult to tell "which is which" on removing the original; and *third*, if the original be lifted up after each puncture, it can never be exactly replaced, and thus the relative position of points will be changed.

For practice in these operations of copying, the student can apply them to any of the simple figures already drawn, or to letters. It is often convenient, moreover, to construct the outlines of ornamental letters and corner-pieces from tracing-paper copies.

Once more, copying may be done by means of what is called transfer paper, which is of an unctuous character, and is placed between the paper containing the given drawing and the blank sheet to which it is to be transferred. The lines of the drawing being then followed with a hard point, an impression of them will be made by the transfer paper upon the blank sheet. This paper is black, blue, red, or green as desired. This assortment is useful for the purpose of transferring colored lines; such as are sometimes added to drawings to indicate alterations, or additions. Thus on a drawing representing a structure as originally designed, wholly in black lines, *alterations* of the original might be in blue lines, and *additions* to it, in red lines; and these it might be convenient to transfer at once in their proper colors, in making copies.

EXERCISES.—1. Having a drawing on a scale of 3 ft. to an inch, find the scale necessary to use in making the copy *three-fourths* as large, and make the copy.

2. Having a drawing on a scale of 5 ft. to 1 inch, and wishing to make a copy $\frac{3}{4}$ as large, *i. e.*, half as large again, what scale must be used, expressing it in feet to the inch, or in inches to the foot?

3. Construct from measurements the plan of the room you occupy; or of the principal floor of your house.

4. Draw the ground-plan of a dwelling containing parlor, dining-room, library, hall, kitchen, and wash-room, with suitable closets; also which *shall* have back stairs, but *shall not* make a passage from kitchen to dining-room through the china-closet.

5. Having the ground-plan of a house, the length of the body parallel to the road, hall in the middle, and two rooms each side, kitchen wing, etc., in the rear, and rooms 12 ft. \times 14 ft.; *design* enlargements by means of extra large bay windows, a tower, or otherwise, and draw the *additions in red lines*. Also throw two of the rooms into one, and indicate the consequent *alterations by blue lines*.

6. Design the plan of a suburban railway station, with the necessary baggage, telegraph, ticket, and toilet rooms, also the outside platforms, with appropriate hack and express stands.

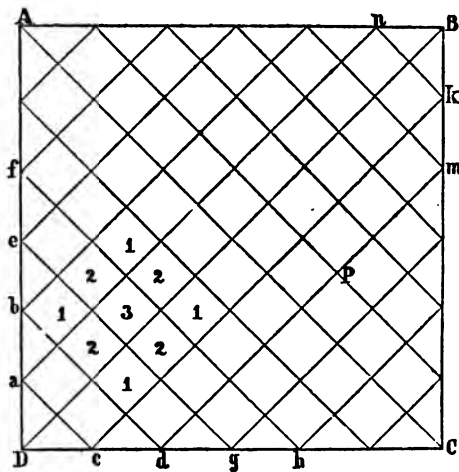
CHAPTER II.

CONSTRUCTION OF GROUPS OF REGULAR FIGURES.

209. The following figures may be taken as geometrical designs for pavements, for oil-cloth patterns, or for printed fabrics. They are meant to be tinted, in two or more shades, by the student, as will presently be explained. The tinting is not shown in the figures, in order that the construction may be made more apparent. Many figures of this kind afford the best of practice in accuracy of construction, as will readily appear on trial. Before commencing them, therefore, let the student have his instruments in perfect order, and make only the finest lines, through points exactly located. We will now, for each figure, describe, *first*, its *construction*; and *secondly*, its *execution*.

210. Fig. 30 represents a *pavement of square tiles*, set diagonally.

FIG. 30.

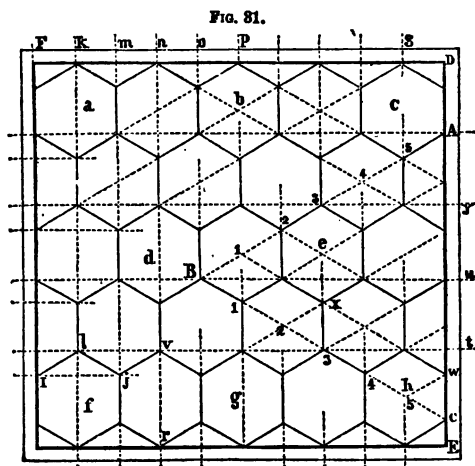


In practice, if ABCD be a floor of given size, and of any rectangular shape, the diagonal of each tile will be a common divisor of two adjacent sides of the floor.

If no such common divisor can be found which will give the tiles an appropriate size, the margin can be filled out with a border, as seen in Fig. 31, or other fractional parts of tiles than halves can be taken. In Fig. 30, having divided the adjacent sides, AD and DC, into equal parts, draw diagonal lines in both directions through each point, and through the points formed by construction on AB and BC, till the whole space, ABCD, is filled.

211. *As regards execution*, let the pencil lines thus drawn remain untouched, if they are so fine as not to show, when the tint is finished; otherwise, ink them with very pale ink, and then carefully erase the pencil lines, remembering that there is danger of injuring the surface of the paper by severe use of the rubber. Then tint over the entire square, ABCD, with a pale tint. When that is dry, tint over alternate stripes, as *abcd* and *efgh*, and when these are dry, tint the alternate stripes in the contrary direction, as *DanB* and *cdkm*. This order of tinting in intersecting stripes will, without any effort at arrangement, produce the combination indicated by the figures, where 3 is the dark centre of a group of nine squares, of which the four corner squares, marked 1, have but one tint, while those marked 2 have two tints. A pavement of rhombuses may be similarly tinted.

212. Fig. 31 is a *pavement of hexagonal tiles*. The width, *ij*,



of a tile, equals the side, $il \times \sqrt{3}$, hence the length, iw , of a floor containing a given number of tiles in each row equals $il \times \sqrt{3} \times$ the number of hexagons in the row. Conversely, if, in a floor of

given length, we wish to place a given number of hexagons, the side, il , of a hexagon will be found by dividing the length by the number of hexagons $\times \sqrt{3}$, i.e. let L = length of floor, n = number of tiles in a row.

$$\text{Then } il = \frac{L}{n\sqrt{3}}.$$

For the width, DE , of the floor, it is shown in the figure that when the whole floor is evenly filled, the width, DE , is divided into equal parts, DA , Ay , etc., each equal to $\frac{2}{3} il$, or $\frac{2}{3}$ of the diameter of the circumscribing circle of a tile, leaving one space, tE , equal to the whole diameter of the same circumscribing circle.

The following, therefore, is *one method of constructing* Fig. 31. Lay off from a corner, as F , Fm , mo , etc., each equal to the side, $il \times \sqrt{3}$, in this case .433 inches, and at these points draw fine vertical lines. These lines will contain some of the vertices and vertical sides of the hexagons. Similar vertical lines, as kf , midway between those just drawn, will contain the remaining vertices and vertical sides. Next, draw tE at a height above E , equal to the extreme diameter of a tile,—i.e. equal to twice il —and divide the remaining space, tD , into equal parts, each equal to $\frac{2}{3}$ of il , and through these points of division draw horizontal lines which will intersect the vertical lines at F , m , o , etc., in vertices of hexagons. With the help of these lines, if they are accurately drawn, the hexagons can readily be completed with the 30° triangle, observing, also, that the oblique sides form portions of groups of oblique parallel lines, as AB .

Another method of construction is as follows. On a line, AC , on which the side of a tile can be laid off an exact number of times, construct an equilateral triangle, ABC , and on each of its sides lay off the side of a hexagon. Through the points 1—1, 2—2, etc., thus formed, draw lines parallel to the sides of the large triangle, and by comparing the full and dotted portions shown in the figure, it will be seen how to form hexagons by the intersections of these lines.

213. In *execution*, tint the whole figure lightly, then tint every second hexagon in every second row with a dark tint, i.e. the hexagons a , b , c — d , e — and f , g , h . Or, make every *third* hexagon dark, in *every* row, as h , vr ,—2, etc.— d , etc. Thus, each dark tile will have six light ones around it. Since tiles of the same shade touch each other in this figure, they may be separated by lines ruled with the tint of the dark tiles.

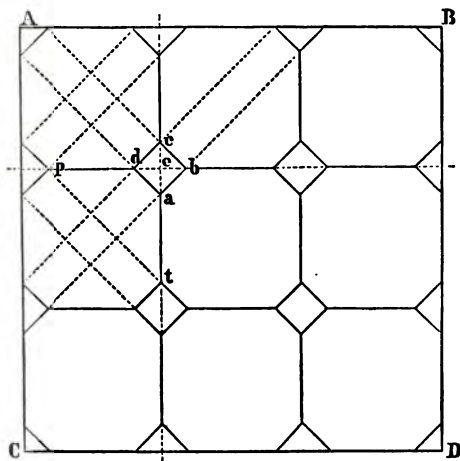
214. Since a hexagon may be decomposed into six equilateral triangles, a surface may be divided into equilateral triangles, and as will readily be seen by trial, these triangles can be arranged otherwise than in hexagonal groups.

215. Equilateral triangles, squares, and hexagons, are the only *regular* polygons with which an area can be filled without leaving open spaces between the figures.

This statement is proved as follows. All the angular space about a point is equal to four right angles. In Fig. 30, at any point, as p , four right angles—one from each of four squares—have p for a common vertex. In Fig. 2, at any point, as x , three angles of hexagons, each being 120° , have a common vertex; or, at the same point, six angles of equilateral triangles, each angle being equal to 60° , have their common vertex. The word *regular* is here used in the geometrical sense as referring to figures whose *sides* and *angles* are equal. The remaining designs of this chapter are regular in the sense that they consist of equal figures, of one or more forms, regularly arranged.

216. Fig. 32 is composed of octagons and small squares. The

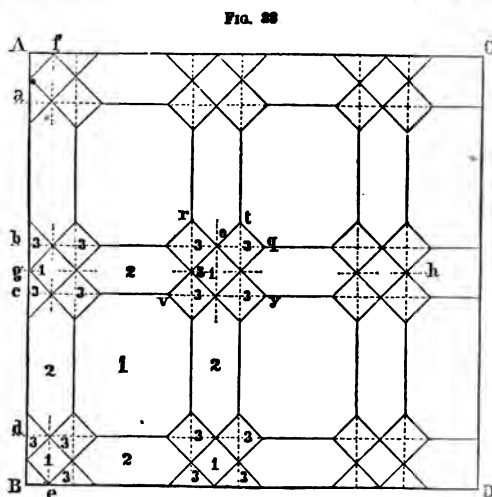
FIG. 32.



dotted lines indicate fully enough the manner of construction, and show that *every* point of each small square need not be laid off with compasses.

In tinting this figure the small squares are made dark, and the octagons are separated by lines of dark tint.

217. Fig. 33 is made up of octagons, elongated hexagons, and



groups of five small squares each. Take the width, Bd , in one pair of dividers, and dc in another, and lay these off alternately on two sides of the area to be paved. Through the points so found, draw lines parallel to the sides of the area. Their intersections will be the centres of the four outer squares of each group.

Other lines, parallel to those just mentioned, and midway between them, as at g and o , determine, by their intersections with the first group of parallels, the four corners, as o , s , etc., of each of the central squares.

Through these points draw diagonals, as tv and ry , and connect their extremities, as at tq , which will complete the construction.

218. The tints are arranged as shown by the figures, with the octagons and central squares lightest, the hexagons intermediate, and the remaining four squares of each group dark. This design makes a handsome plate on tinted paper, with the octagons and central squares painted white; the hexagons, slate color (dull blue); and the remaining squares perfectly black: or with other well contrasted colors, laid on thick.

219. The construction of figures 34, 35, and 36 is evident on inspection. They may be tinted in clear colors, or in neutral tints, as well as with ink.

In dividing up a line, as Ae , Fig. 34, into alternate large and small parts, another way is to lay off the sum of both, as Ae , ce ,

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etc., and then from A , c , e , etc., to lay off the small parts, as Ab , cd , etc.

FIG. 84.

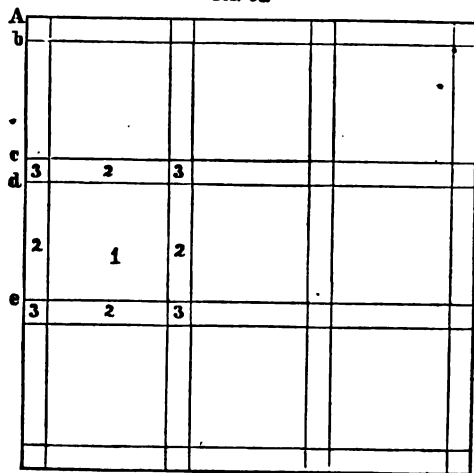


FIG. 85.

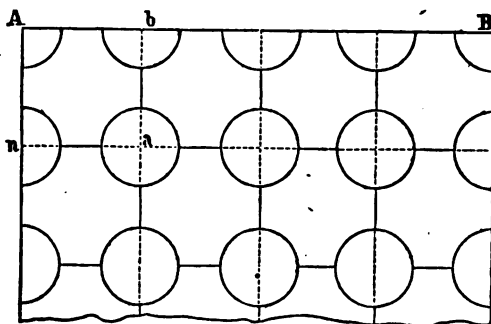
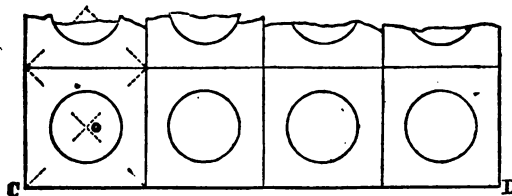


FIG. 86.



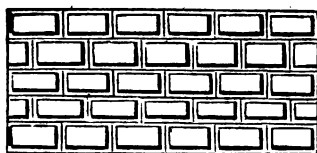
Also in the execution of the same figure the narrow stripes may be ruled in perfectly black lines, nearly as wide as the space, cd .

This will give good practice in ruling wide lines, which require very thick ink. Further practice of the same kind may be had by surrounding any of the designs, as Fig. 34, when done on tinted paper, with contiguous bands, the inner one white, the next slate color, and the outer black, for example, each to be ruled with the drawing-pen after the preceding one has become dry.

220. Some additional geometrical designs for surface decoration are shown in Figs. 37 to 41 inclusive.

Fig. 37 represents a wall front, decorated by a slightly project

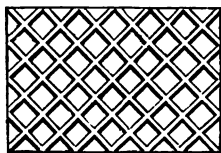
FIG. 37.



ing rectangular portion of each stone, and nearly equal to the full size of the stone. The construction is obvious on inspection.

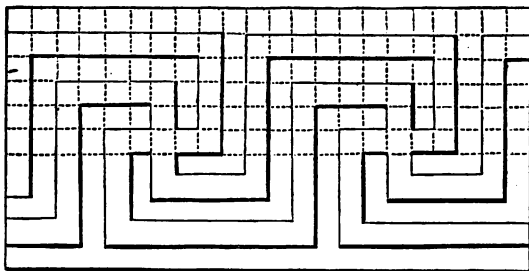
Fig. 38 is a reticulated design. Divide the sides of the bound-

FIG. 38.



ing rectangle into equal parts, to guide in drawing the pairs of diagonals. After drawing the diagonals, such parts of them should

FIG. 39.



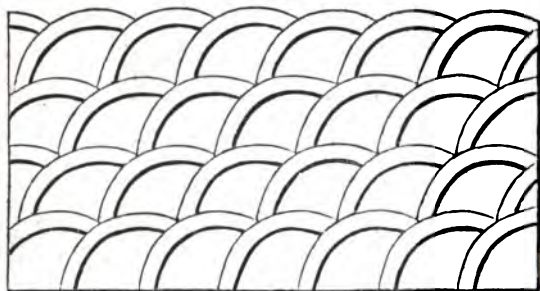
be inked as form the sides of the small detached squares composing the design. In all these designs, certain lines are shaded to

represent the effect of an oblique action of the light upon the raised portions.

Fig. 39 is a simple example of a class of designs or architectural bands, in cornices, etc., and called Grecian. As appears by inspection, they are easily constructed by filling the proposed area with equal squares, whose sides are equal to the width of the bands and included spaces which form the proposed pattern.

Fig. 40 is a design in circular work. The end of each outer arc

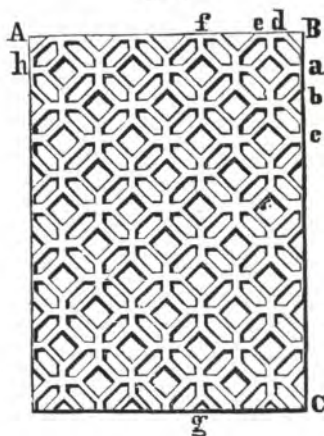
FIG. 40.



of a pair, is the centre for one pair of the concentric arcs. Such a design is applicable in ornamental glass panels, etc.

Fig. 41 presents a design of considerable complexity, affording

FIG. 41.



excellent practice in perfect precision in the construction. The design consists of a web of octagons in two sets.

Lay off on each side, as AB and BC, the proposed size of the octagons, as Bb. Then bisect these spaces, giving points as *a*, *c*, and *f*, belonging to the second set of octagons. Next, construct with care one perfect octagon, as *ed a*, locate on AB and BC other points, as *e* and *d*, and project them across upon the opposite sides. Points will thus be established in the centre-lines of each of the four sets of pairs of parallels. These parallels may next be drawn, in the finest lines, as at *fg*, *ah*, and in the two diagonal directions, as at *d* and *e*. By then inking such portions of these parallels as are shown in the figure, the design will be completed.

In all cases where such perfect accuracy is demanded as is necessary in this design, it is better to locate guiding-points, as *a* and *h*, at both ends of the lines, than to trust to the triangles alone, as used in drawing parallels.

Figs. 37, 38, and 41, may advantageously be made by the student on a larger scale than is here shown.

While the preceding wood-cuts may serve the learner as copies for construction, plates VI. and VII. will not only afford a variety of beautiful designs,* but will answer better as models of fine execution, for all plates of similar work, whether done in ink or in colors.

The student who desires to execute some of the foregoing designs in colors, will do well to postpone the work till after reading the section on "harmony of color" in Div. IV., and "CHEVREUL on Colors," in connection therewith, if he wishes to go into the subject of color decoration extensively, in accordance with the present general demand for the artistic use of color.

* Selected from the French of DELAISTRE, *Cours de Dessin Linéaire*; a work of varied and valuable contents, which this one selection may well induce every student of drafting to add to his library.

CHAPTER III.

GENERALLY USEFUL ELEMENTARY PLANE GEOMETRICAL PROBLEMS.

§ 1.—*Fundamental Problems.*

A.—Projections.

PROB. 1. *To locate a Point at given distances from two given points.* (Fig. 42.)

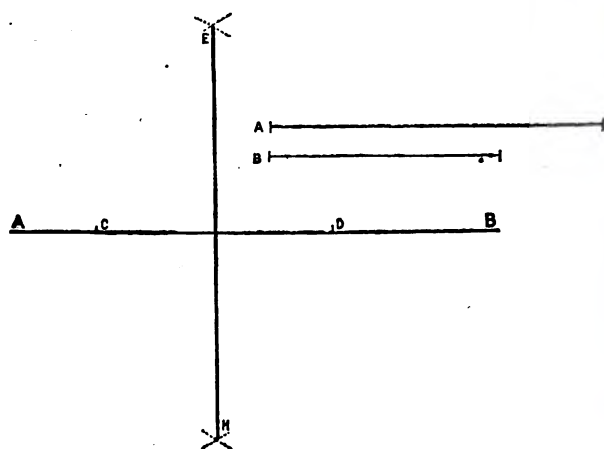


FIG. 42.

Let B and c be the given points, and a and b the given distances.
1st. Take up the distance a as a radius, and, with B as a centre,

describe a short arc near where it is supposed the required point will fall.

2d. With the distance b as a radius, and c as a centre, mark the point e where this radius intersects the arc previously drawn. Then e will be the required point.

Principle.—A point which is in each of two lines must be at their intersection.

Remarks.—(a) By using *both* the above radii at *each* of the points B and c , and by describing arcs with *each* radius on *both* sides of the line Bc, four points can be located at the given distances from the given points.

(b) When the given distances are *equal*, two points, as C and D, Fig. 43, can be located *equidistant* from two given points, as A and B.

PROB. 2. *To construct a Straight Line, every point of which shall be equidistant from two given points.* (Fig. 43.)

Let A and B be the given points.

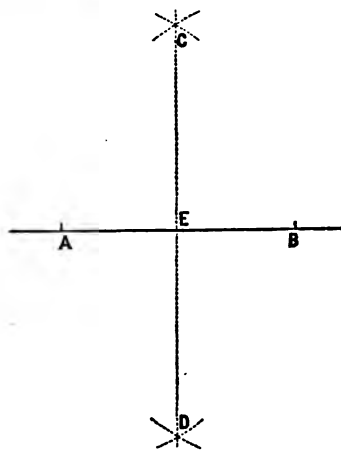


FIG. 43.

1st. By Prob. 1, construct any two points, as C and D, equidistant from A and B.

2d. Join C and D by a straight line, which will be the line required.

Principle.—Since a straight line nowhere changes its direction, it follows that, in passing along a straight line, joining C and D, Fig. 15, which are equidistant from A and B, we cannot approach or leave A any faster than we approach or leave B. Therefore, every point of a straight line which joins two points, which are equidistant from two fixed points, is equidistant from the two fixed points.

Example.—Make the construction when C and D are the given points.

PROB. 3. To locate Two Points on a given line, and equidistant from a point not on that line. (Fig. 44.)

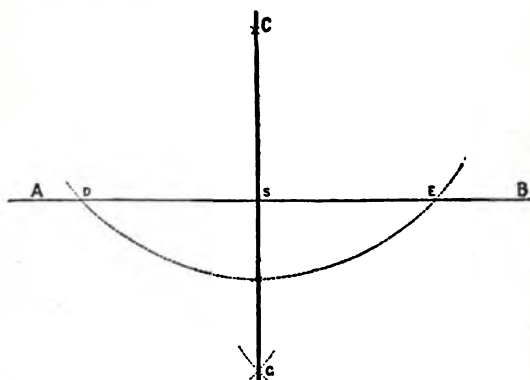


FIG. 44.

Let AB be the given line, and C the given point.

1st. Take any convenient radius in the compasses, then—

2d. With C as a centre, describe an arc, as *de*; and the points *d* and *e* will be equidistant from C.

Principles.—1°. All points on the circumference of a circle are equidistant from its centre.

2°. A point—as *d*—which is at once in two lines, is at their intersection.

Remark.—If the required points were to have been at a given distance from C, that given distance would have been taken as the radius, in the first step of the solution.

PROB. 4. *To make any number of Circular Arcs of a given radius pass through a given point.*

PROB. 5. *To draw a Circle of given radius through two given points.* (Fig. 45.)

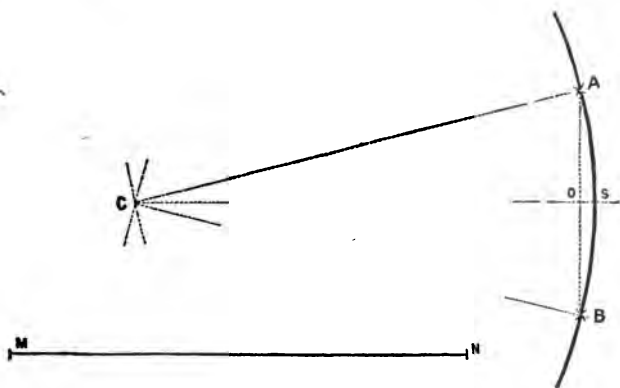


FIG. 45.

Let A and B be the given points, and *mn* the given radius.

1st. By Prob. 1, construct the point C, equidistant, by the distance *mn*, from A and B.

2d. With *mn* as a radius, take C as a centre, and describe an arc which will pass through A and B as required.

Principles.—See Probs. 1 and 3.

Remarks.—(a) If the two given points and the centre are on the same straight line, the former include the diameter of the required circle, and the required centre will be the middle point of this diameter.

(b) If at *o*, the middle point of the chord AB, a perpendicular were to be erected, it would also contain the centre; on this—

Principle—The centre of an arc is in a perpendicular to the chord of that arc, at the middle point of the chord.

PROB. 6. *To draw an Arc through three given points, which are not situated in the same right line.*

Let a , b , and c , Fig. 46, be the given points.

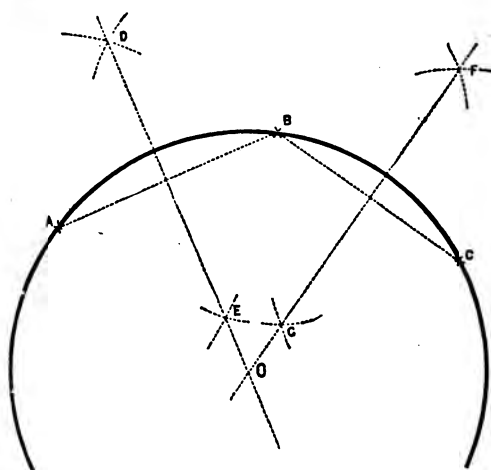


FIG. 46.

1st. By Prob. 2, construct a line, ed , every point of which shall be equidistant from a and b .

2d. Likewise, construct a line, fg , every point of which shall be equidistant from b and c .

3d. With O , the point of intersection of these lines, as a centre, and with the distance to either of the three given points as a radius, describe an arc. This arc will pass through all three of the given points.

Principles.—1°. See Prob. 2.

2°. Through three points, not in the same straight line, but one circle can be made to pass.

PROB. 7. *To find the Centre of a given Circle or Arc, when the radius is known.*

PROB. 8. *To find the Centre of a given Circle or Arc, when the radius is not known.*

B.—Intersections.

a.—Problems of Points and Straight Lines.

PERPENDICULARS.

PROB. 9. *To draw a Line perpendicular to a given line. (Fig. 47.)*

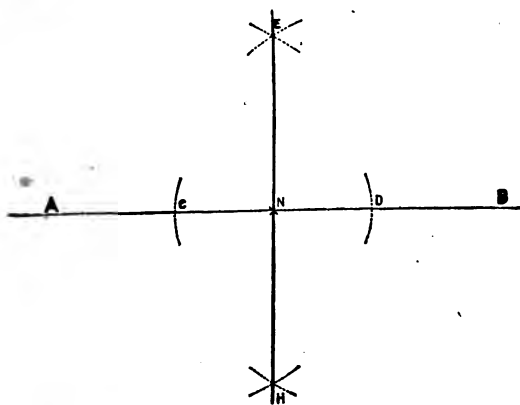


FIG. 47.

Let AB be the given line.

1st. Take any distance, cd , anywhere upon the line AB.

2d. By Prob. 2, construct a line, eh , every point of which shall be equidistant from c and d , and it will be the required perpendicular.

Principle.—Since, in moving on eh , we approach or recede from A and B equally, it follows that in this motion we do not move at all in the direction AB or BA. Hence the direction of eh is *independent* of that of AB. But when the directions of two lines are thus independent, those lines are said to be at right angles, or perpendicular to each other.

PROB. 10. *To draw a Perpendicular to a given line, at any given point on that line. (Fig. 47.)*

Let AB be the given line, and n the given point.

1st. On each side of the given point n , set off an equal distance — $nc = nd$.

2d. The remaining steps, and the principle, are the same as in the last problem.

PROB. 11. *From a point, out of a given line, to draw a Perpendicular to that line.* (Fig. 44.)

Let C be the given point, and AB the given line.

1st. By Prob. 3, locate two points, as d and e , equidistant from C.

2d. By Prob. 1, locate a point, as g , equidistant from d and e , and the line Cg will be perpendicular to AB.

Principles.—See Prob. 10, and the other problems just referred to.

PARALLELS.

PROB. 12. *Through a given point, to construct a Parallel to a given line.*

Let AB be the given line, and C the given point.

1st. By Prob. 11, let fall a perpendicular Cd from C upon AB.

2d. By Prob. 10, erect a perpendicular to AB at any point, a .

3d. Lay off the distance Cd, upon ae , from a .

4th. Join b , the point thus found, with C, and the line bC will be the required parallel.

Principle.—Parallel lines are everywhere equidistant.

PROB. 13. *To draw a Parallel to a given line, and at a given distance from it.*

Remark.—After becoming familiar with ruler and compass constructions of perpendiculars and parallels, the student may construct them with the triangles (119, 120).

b.—Problems on the Circle ; analogous to the preceding.

To locate points, or a line everywhere equidistant from two points on a circle, is to find a diameter of it.

When a line is perpendicular to a curve at any point, it is perpendicular to the tangent at that point. Such a perpendicular is called a *normal*. Every radius of a circle is normal to its circumference. Therefore, to draw a normal from any point out of the circumference, simply draw a line through that point and the centre of the circle.

A parallel to a circle is a *concentric circle* ; that is, a new circle having the same centre as the given circle.

C.—Tangencies.

a.—Tangent Straight Lines.

PROB. 14. *To draw a Tangent at a given point on the circumference of a circle whose centre is known.* (Fig. 21.)

Let the circle whose centre is C be the given circle, and let A be the given point of tangency.

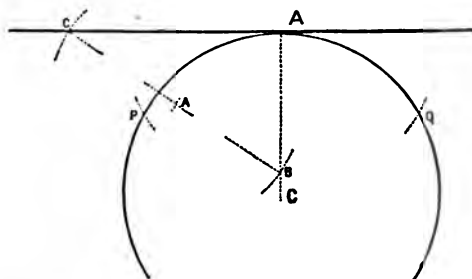


FIG. 48.

1st. Draw the radius CA , and produce it.

2d. Draw, by Prob. 10, Ac , perpendicular to AC at A , and it will be the required tangent.

Principle.—The tangent to a circle is perpendicular to the radius at the point of tangency.

PROB. 15. *To draw a Tangent at a given point of an arc or circle whose centre is not given.*

Let BAC be the given arc, and let A be the given point on the arc. (Fig. 22.)

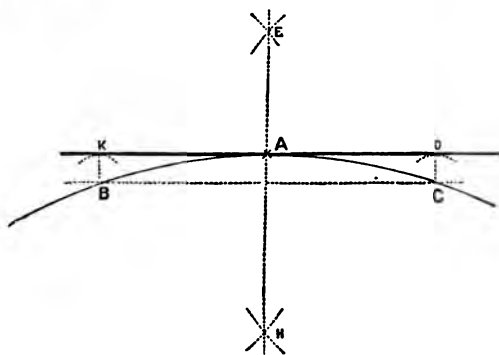


FIG. 49.

1st. Set off on the arc equal distances, AB and AC , on each side of the point A .

2d. By the constructions already given for drawing a line

through a given point and parallel to a given line, draw, through A, dA parallel to BC, and it will be the required tangent.

Principle.—The tangent to a circle is parallel to the chord which connects points on the circumference equally distant from the point of tangency, since the tangent and such a chord are both perpendicular to the radius of the point of tangency.

PROB. 16. *To draw a Straight Line tangent to a given circle, and through a given point not in the circumference of that circle.*

Let C be the centre of the given circle, and let A be the given point. (Fig. 50.)

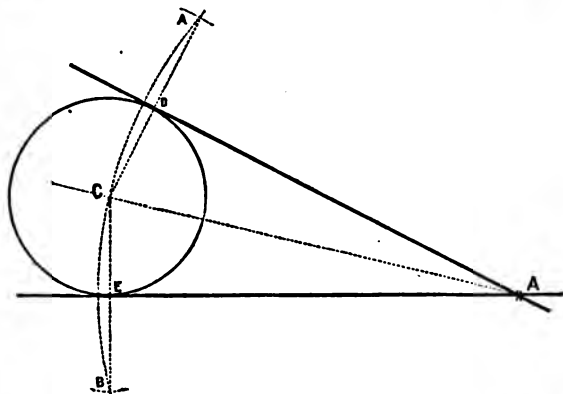


FIG. 50.

1st. With AC as a radius, describe an arc, ab , from A as a centre.

2d. With C as a centre, and with the diameter of the given circle as a radius, describe small arcs cutting the arc ab , at the points a and b .

3d. Draw the chords Ca and Cb , and note the points, d and e , where they cross the circle.

4th. Draw dA and eA , and these lines will be the required tangents.

Principles.—1°. The radius through the middle point of a chord— Ca or Cb —is perpendicular to that chord.

2°. The radius of a circle is perpendicular to the tangent at its extremity.

b.—Tangent Circles.

PROB. 17. *To draw a Circle of given centre, tangent to a given straight line.*

PROB. 18. *To draw a Circle from a given centre, and tangent to a given circle.*

Remark.—Circles on opposite sides of their common tangent are tangent to each other *exteriorly*. If on the same side, they are tangent *interiorly*.

PROB. 19. *To draw a Circle, of any given radius, tangent to a straight line, at a given point on that line.*

PROB. 20. *To draw a Circle, with a given radius, and tangent to a given circle, at a given point.*

D.—Development.

a.—Equivalent Lines and Arcs.

PROB. 21. *To bisect a given Distance on a given line.*

Let AB be the given distance. (Fig. 43.)

1st. By Prob. 2, construct a line, CD, every point of which will be equidistant from A and B.

2d. Note the point E, the intersection of CD with AB, which will bisect AB as required.

Principle.—Since every point of CD is equidistant from A and B, its intersection with the line joining those points will bisect the distance between the same points.

Remarks.—(a) To facilitate accuracy in the construction of this and all problems, where similar operations are performed, it is desirable, *first*, to have the arcs cut each other nearly at right angles; and *second*, to have the points C and D at a considerable distance from AB. To secure both these results, we cannot do better than to take AB—the given line itself—as the radius for describing the intersecting arcs.

(b) It is sufficient to make the arcs at C and D quite short, as a little attention will enable one to judge, in advance, of the position of their intersection. Also, in this problem, it is not necessary to draw a line from C to D, but only that point of it which bisects the given line AB at E.

(c) This problem reduces to that case of Prob. 1, in which the required point is on a straight line joining the given points.

PROB. 22. *To develop a Circle or Arc, upon a straight line, or upon another arc, A, of different radius.*

1st. Divide the given circle or arc into any number of parts, not necessarily equal, but so small that their chords shall be substantially equal to the subtended arcs.

2d. Lay off these chords successively on the straight line, or the second arc, A. The whole distance thus found will be equal to the given circle or arc, and is called its development.

Principle.—The whole is equal to the sum of all its parts.

Remarks.—(a) If the arc A be of nearly the same radius as the one to be developed, the chords laid off upon it may be quite large without involving appreciable error.

(b) The construction of a straight line, equivalent to a given arc, is commonly called the rectification of the arc.

b.—Proportional Lines, and the Construction of Distances.

PROB. 23. *To divide a given Line into Two Parts, proportional to two other given lines.*

Let AB be the given line, which is to be divided, and let *cd* and *ef* be the other given lines. (Fig. 51.)

1st. Through A, one extremity of the first given line, draw a line, AF, in any direction, not coinciding with AB.

2d. From A, lay off AD equal to *cd*, and from D, lay off DF equal to *ef*.

3d. Connect F with B, the opposite extremity, from A, of the first given line.

4th. Through D, draw *Dp* parallel to FB, and at *p*, the line AB will be divided into parts proportional to the given lines *cd* and *ef*.

Principle.—A line drawn parallel to the base of a triangle—*Dp*, parallel to FB—divides the other sides proportionally.

Remarks.—(a) Practically, it is well to make the angle at A one of about 60°.

(b) If AB were of *very* great length, compared with the sum of the other given lines, it would be necessary to find two parts of some line less than AB, which should be proportional to *cd* and *ef* and then to divide AB into parts proportional to these auxiliary proportionals.

(c) If the problem merely required *any* two lines proportional to *cd* and *ef*, then we should make AB and AF each of indefinite length, and should draw through D and F parallels in any direction. These parallels would cut off parts of AB, proportional to *cd* and *ef*.

PROB. 24. *To divide a given Straight Line into Parts propor-*

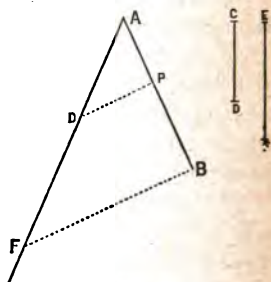


FIG. 51.

tional to any number of other lines ; or to the parts into which any other line is divided.

This can be solved by the student in the same manner as the last problem.

Second Method.—Fig. 52. Let AB be a straight line with the given parts set off upon it, and let gk be the given line to be divided.

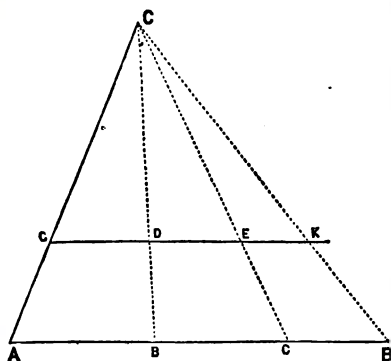


FIG. 52.

1st. Join Ag and Bk , and produce those lines till they meet at some point, C .

2d. From C draw lines to the several points of AB , and they will divide gk as required.

Principle.—In any triangle, if a line be drawn parallel to the base, lines drawn from points in the base to the opposite vertex will divide the base and the parallel line proportionally.

Remark.—The problem can readily be solved, also, if one of the required parts be given.

PROB. 25. *To divide a Straight Line into a prime number of Equal Parts.*

Let AB be the given line, and let it be required to be divided into seven equal parts. (Fig. 53.)

1st. Through either end of the line, as A , draw a line, AC , in any direction.

2d. Take any convenient distance in the dividers, and, commencing at A , set it off seven times on AC .

3d. Join the points C and B .

4th. Through all the points of division on AC , draw lines parallel to CB , and they will cut the line AB in points, as n , so as to divide AB into equal parts.

Principle.—In any triangle— ABC —if a line—as on —be drawn parallel to the base— BC —it will divide the other two sides— AB and AC —proportionally.

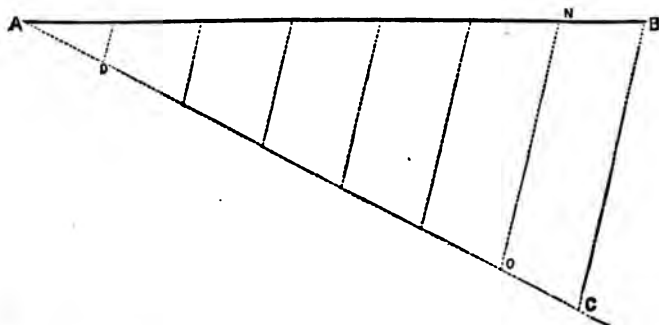


FIG. 53.

Remarks.—(a) Practically, it is well to arrange the figure in some way, so that, as shown in the diagram, the angles, as at o and at n , shall not be very acute.

(b) A straight line may be divided into any composite number (61) of equal parts by the same construction, as well as by those already given. It may be divided into two equal parts by Prob. 21 or Prob 23.

PROB. 26. *To construct a Fourth Proportional to three given lines.*

Let a , b , and c be the given lines. (Fig. 54.)

1st. Draw any two indefinite lines, Ae and Ac , making any angle with each other.

2d. On either of these lines, as Ac , lay off Ab , equal to the given line a , and bc or Ad' , equal to the given line b .

3d. On the other line, Ae , lay off Ad , equal to the given line c ; or make $bc' = c$.

4th. Draw bd , or bd' .

5th. Make ce parallel to bd , and de will be the required fourth proportional; or make $c'e'$ parallel to bd' , and $d'e'$ will be the same fourth proportional.

Principle.—The same as in the last problem.

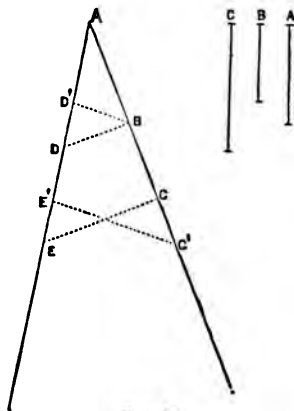


FIG. 54.

Remarks.—(a) A fourth proportional, to three given lines, is a line, such that the first line is to the second as the third is to the fourth.

(b) A third proportional, to two given lines, is a line, such that the first is to the second as the second is to the third, *i. e.* the second is a mean proportional between the first line and the third, or required line.

PROB. 27. *To construct a Third Proportional to two given lines.*

Let a and b , Fig. 55, be the given lines; the latter, the mean proportional.

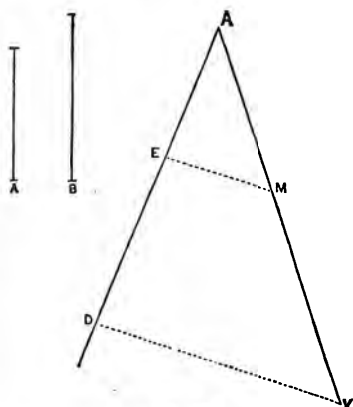


FIG. 55.

1st. Draw any two lines, Ad and Av , making any angle with each other.

2d. On either of these lines, as Ad , lay off the distances Ae and ed , respectively equal to the given lines a and b .

3d. On Av , lay off Am , also equal to the given line b , the mean proportional.

4th. Draw em .

5th. Draw dv , parallel to em , and mv will be the required third proportional to a and b .

Principle.—The same as in the last problem.

PROB. 28. *To find a Mean Proportional to two given lines.*

Let a' and b' be the given lines. (Fig. 56.)

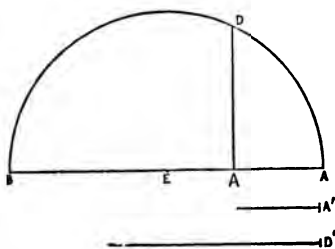


FIG. 56.

1st. Draw any indefinite straight line, ab , and on it mark some point, as A .

2d. Lay off the line a' on ab , and on either side of A , as at Aa .

3d. Lay off the line b' , on the opposite side of A , as at Ab .

4th. Bisect ab at e .

5th. On ab , as a diameter, describe the semicircle adb .

6th. From A , erect the perpen-

dicular Ad ; note the point, d , where it crosses the semicircle, and Ad will be the required mean proportional.

Principle.—The perpendicular, drawn from any point on a semicircle, to the diameter, is a mean proportional between the two segments— Ab and Aa —into which it divides the diameter.

PROB. 29. *To divide a given line medially; that is, into two parts, such that the whole line shall be to the greater part as the greater part to the lesser part.*

Let AB be the given line. (Fig. 57.)

1st. At either extremity of the line, as B , erect a perpendicular, BC , and make it equal to half of AB .

2d. With BC as a radius, and C as a centre, describe the arc DBE .

3d. Draw ACD .

4th. With A as a centre, and with AE as a radius, describe an arc, and at F , where it cuts AB , AB will be divided in the manner required.

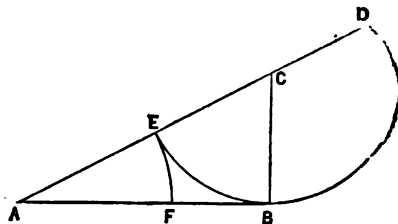


FIG. 57.

Principles.—1°. A tangent— AB —is a mean proportional between the secant— AD —and its external segment— AE .

2°. Quantities which are in proportion will be in proportion by division; that is, $AE : AB :: FB : AE$.

3°. Quantities which are in proportion will be in proportion by inversion; that is, $AB : AF :: AF : FB$.

Remark.—This division of AB is also called *division in extreme and mean ratio*, at the point F .

c.—Angles.

PROB. 30. *To construct an Angle equal to a given angle, and at a given point on a given line.*

Let BAC be the given angle, and D the given point on the given line, DE . (Fig. 58.)

1st. With A as a centre, and any convenient radius, describe an arc, BC .

2d. With D as a centre, and the same radius as before, describe an arc, FE .

3d. Take the chord of BC as a radius, and, with E as a centre, describe an arc cutting EF at F .

4th. Draw FD , which will make FDE the required angle.

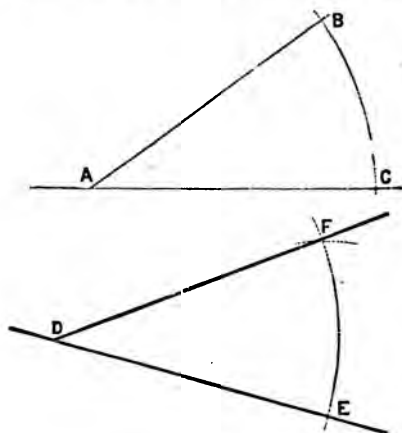


FIG. 58.

Principles.—1°. Equal chords measure equal arcs, in the same or in equal circles.

2°. Equal arcs, in equal circles, measure equal angles at the centres of those circles.

PROB. 31. *To bisect a given angle.*

Let ABC be the given angle. (Fig. 59.)

First Method.—1st. From the vertex B , lay off any convenient distance, as $Bd = Be$, on both sides of the angle.

2d. By Prob. 1, Rem. *b*, construct a second point, as f or g ,— f , being furthest from B , is preferable on the score of accuracy—equidistant from d and e . B is thus equidistant already.

3d. Draw fB , which will bisect ABC as required.

Principles.—1. The bisecting line of the chord of an arc (de) bisects the arc.

2°. Equal arcs of the same radius subtend equal angles at their common centre.

Second Method.—1st. Through any point, as d , of either side of the given angle, draw a line (not shown) parallel to the other side.

2d. On this line lay off a distance equal to dB , and within the given angle, and the line joining the point so found with the vertex B will bisect the given angle.

Principles.—1°. The angles at the base of an isosceles triangle are equal.

2°. The alternate angles formed by a secant meeting two parallels are equal.

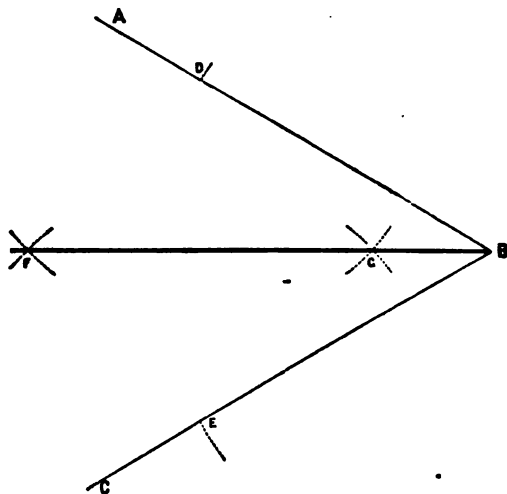


FIG. 59.

3°. Things equal to the same thing are equal to each other.

PROB. 32. *Through a given point, not on a given line, to draw a Line, making a given Angle with the given line.*

Let r be the given point, AB the given line. (Fig. 60.)

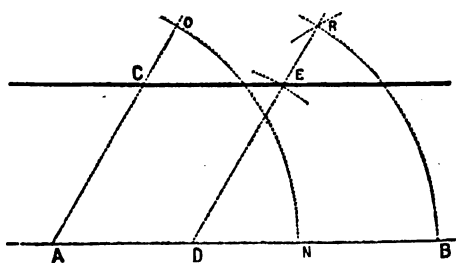


FIG. 60.

1st. At any point, A, of the given line, construct an angle equal to the given angle, by Prob. 30.

2d. By Prob. 12, draw a line, rD , through r , and parallel to oA , and it will be the line required.

Principle.—If a straight line meet two parallels, the opposite exterior and interior angles will be equal.

§ 2.—*Special and Practical Problems.*

PROB. 33. *Upon a given radius, to construct a Circle by points.*

Let OB be the given radius. (Fig. 61.)

1st. Make OC and BD ,

each perpendicular to OB , and equal to it.

2. Divide OB into any convenient number of equal parts.

3. Divide BD into the same number of equal parts.

4th. Produce CO to A , making $OA = OC$.

5th. From A , draw indefinite lines through the points of division on OB ; and from C , draw lines to the points on BD . These two groups of lines will intersect at points, a, b, c, d , which will be in the circumference of a circle of radius OB .

Remark.—The three other quadrants can be determined in the same way.

Proof.—Suppose the circle to be first drawn, and that the intersecting lines, as $1C$ and aA , have been drawn through points on the circumference. Then, in the triangles $A1O$ and ICD , the angles at D and O are right angles, the angles at C and A are equal, each being measured by half the arc aC , and $AO = CD$. Therefore, the two triangles are equal, and $O1 = D1$; *i. e.* the lines OB and BD are properly divided into equal parts, as in the foregoing construction.

Remark.—If the base of a triangle be divided into equal parts, and if lines be drawn from the points of division to the opposite angle, then any line parallel to the base will be divided into equal parts by these lines—produced, if necessary. Hence, in the figure, lines, as en and CE , and cv , will be divided into equal parts by the same groups of lines that were used in determining points in the circumference. This remark serves to show how to construct a circular arc by points.

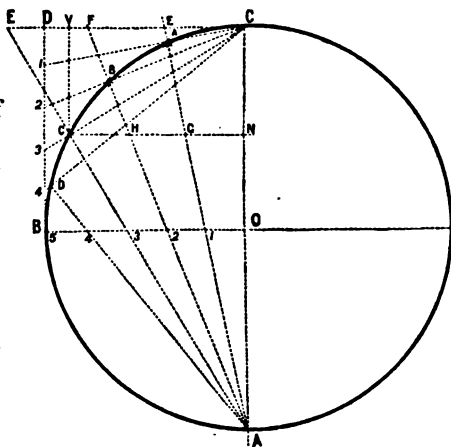
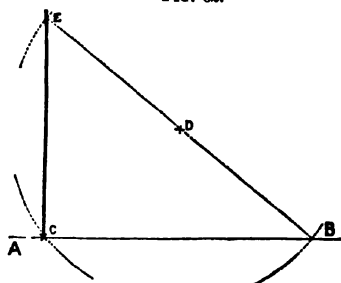


FIG. 61.

PROB. 34. To draw a Perpendicular from a point, at or near the end of a given line.

Let AB be the given line, and c the given point. (Fig. 62.)

FIG. 62.



First Method.—1st. With any point, d , not on the given line, as a centre, and dc as a radius, describe an arc, and note the point, B , where it cuts the given line.

2d. Draw the diameter Be , and note the point, e , where it cuts the arc.

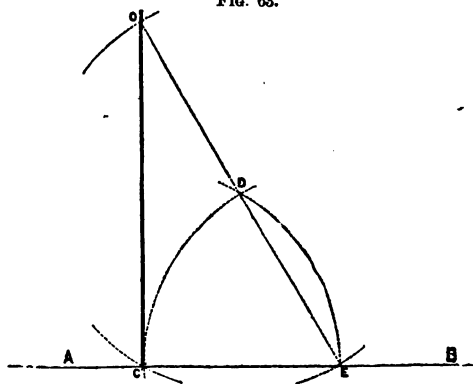
3d. Draw ec , which will be perpendicular to AB , as required.

Principle.—Any angle— ecB —inscribed in a semicircle— ecB —is a right angle.

Remark.—To secure a good construction, the relative position of d , c , and B should not vary much from that shown in the diagram.

Second Method.—Fig. 63. Let AB be the given line, and c the given point.

FIG. 63.



1st. Place the steel point of the dividers at c , extend the pencil point to any point, e , and, with c as a centre and ce as a radius describe an arc, ed .

2d. With e as a centre, and with the same radius, describe the arc cd , cutting the former arc at d .

3d. Draw the line ed , and produce it indefinitely.

4th. With d as a centre, and de as a radius, describe the arc eco , and note the point o .

5th. Draw oc , which will be perpendicular to AB , as required.

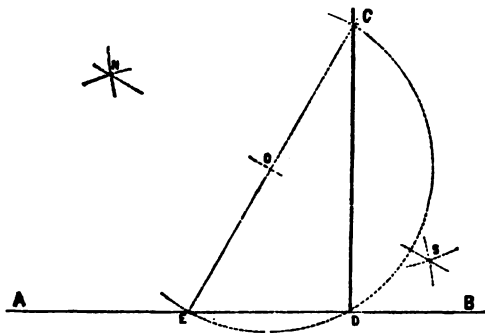
Principles.—1°. The centre— d —of a circle, is equally distant from all points—as o , c , and e —of the circumference.

2°. See the first method.

PROB. 35. *To draw a Perpendicular to a line, from a point nearly over the end of the line.*

Let AB be the given line, and C the given point. (Fig. 64.)

FIG. 64.



First Method.—1st. Mark any point, as e , not very far from where the perpendicular Cd might be supposed to cut AB .

2d. Join this point e with C .

3d. Bisect Ce at o , as shown by the constructions at n and s .

4th. With o as a centre, and $oe = oC$ as a radius, describe the arc edC , and note the point d .

5th. Draw Cd , which will be perpendicular to AB .

Principle.—See Prob. 34.

Second Method.—Fig. 64. Let AB be the given line, and C the given point.

1st. With any point, as B , near to where the perpendicular will cut AB , as a centre, and with BC as a radius, describe an arc, Cde , not shown.

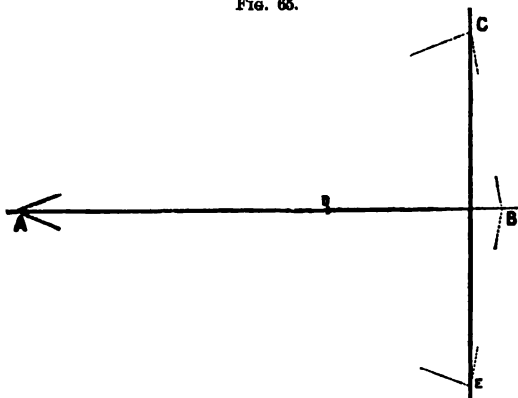
2d. With any other point, as A , as far as convenient from B , as a centre, and with AC as a radius, describe the arc CBe , also not shown.

3d. Join C and e , and Ce will be perpendicular to AB .

Principle.—When two circles— Cde and CBe —have two points

common to each— C and e —the common chord— Ce —joining those points, will be perpendicular to the line— AB —which joins the centres— A and B —of the circles.

FIG. 65.

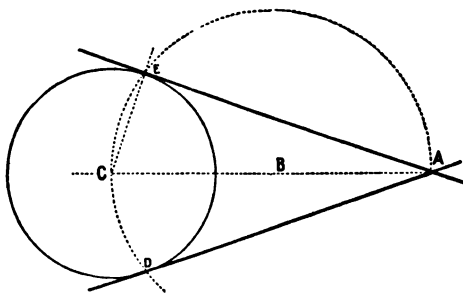


Remark.—The centres, A and B , might be on the same side of Ce , but if they were, the intersections of the two arcs would be less accurately defined than in the construction here given.

PROB. 36. *To draw a Tangent to a given circle, from an exterior point.*

Let C be the centre of the given circle, and let A be the given point. (Fig. 66.)

FIG. 66.



1st. Join A and C .

2d. Find by trial, or by construction, B , the middle point of AC .

3d. With B as a centre, and with BA as a radius, describe the arc deA .

4th. Draw Ae and Ad , and they will both be tangents, answering the given conditions.

Principles.—1°. By drawing Ce , we have: the angle—as CeA —which is inscribed in a semicircle— CeA —is a right angle.

2°. The tangent to a circle is perpendicular to the radius passing through the point of tangency.

PROB. 37. *Having given a side and two adjacent angles of a triangle, to construct the Triangle.*

Let AB be the given side, and C and D the given angles.

1st. On any indefinite line, EF , lay off a distance EF , equal to the given side AB .

2d. At either end of EF , as at E , construct the angle mEn , equal to the given angle C , by Prob. 30.

3d. At F , construct the angle oFp , equal to the other given angle, D .

4th. Produce the sides En , and Fp , till they meet at G ; which will complete the triangle.

Principle.—When a side and its two adjacent angles, the given side and angles, in one triangle are equal to the side and two adjacent angles in another triangle, EGF , the triangles are equal.

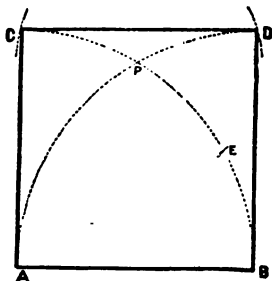
Remark.—If one of the given angles were opposite the given side; find the third angle by making it the difference between 180° and the sum of the other two. Then proceed as above.

PROB. 38. *To construct a Square on a given line.*

Let AB be the given side.

First Method.—(Fig. 67.)

FIG. 67.



1st. AB being laid off on any indefinite line, erect, at A and B , perpendiculars to AB .

2d. On each of these perpendiculars, lay off a distance equal to the given side.

3d. Join the points C and D, thus found, and the square will be completed.

Principle.—A square is a figure of four equal sides, and having four right angles.

Second Method.—(Fig. 67.)

1st. Lay down AB, as before.

2d. With A and B as centres, and with AB as a radius, describe arcs intersecting at *p*.

3d. Bisect either of these arcs, as *pB*, at *e*, and lay off the chord of *pe* on the arcs produced, from *p* to C and D.

4th. Draw CA, CD and DB, which will complete the square.

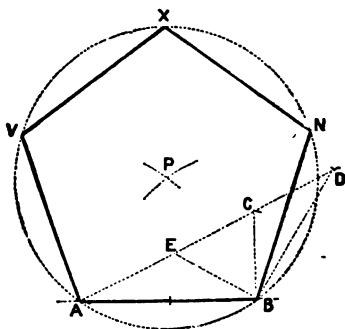
Principles.—1°. See the first method.

2°. Radius equals the chord of 60°.

PROB. 39. *To construct a regular Pentagon, on a given side.*

Let AB be the given side. (Fig. 68.)

FIG. 68.



1st. At either end of AB, as B, erect a perpendicular, BC, equal to half the given side.

2d. Draw ACD and make CD equal to CB.

3d. Join B and D. BD is the radius of the circumscribed circle of the required pentagon; hence,

4th. With BD as a radius, and with A and B, taken successively as centres, draw arcs meeting at P.

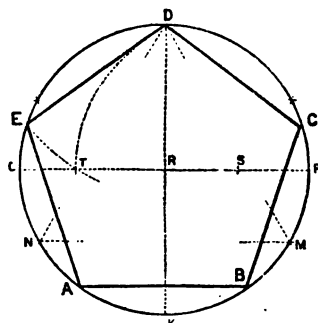
5th. Describe a circle with P as a centre, and PA as a radius, and apply the chord AB five times to the circumference; which will give the pentagon ABN XV.

Principles.—1°. By making $CE = CD = CB$, AD is divided me-

dially at E. Also $AB=ED$. Hence in the triangles AEB and ABD, $AE : AB :: AB : AD$. Also, the angle BAE is common to both. Hence these triangles are similar, and $AE : EB :: AB : BD$, or by alternation $AE : AB :: EB : BD$.

2°. As there is but one division in extreme and mean ratio for any one line, any two lines so divided, are proportionally divided. Hence, observing that AB is thus divided, by taking AE for its larger segment, and that ro (Fig. 69) is likewise divided at t , (see

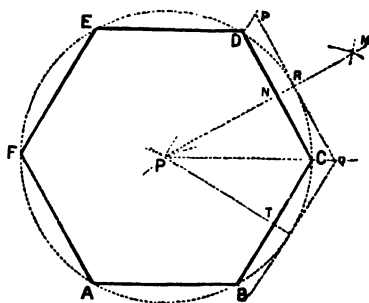
FIG. 69.



Prob. 42,) and that $rD=ro$, we have, supposing Figs. 68 and 69 to be equal pentagons, $AE : AB :: tr : rD$. Hence, from the last two proportions, $EB : BD :: tr : rD$. Hence the triangles EBD and trD are similar. But $tD (=DE)$ Fig. 69, $=ED (=AB)$ Fig. 68, hence these triangles are not only similar, but equal. Therefore $BD (=rD)$ is the radius of the circumscribing circle of the pentagon of which $ED=AB=tD$ is a side.

PROB. 40. *To construct a regular Hexagon on a given line.*
Let AB be the given line. (Fig. 70.)

FIG. 70.



First Method.—1st. Take AB as a radius, and describe arcs from A and B as centres, intersecting at P.

2d. With the same radius, describe a circle, having P for its centre. Then proceed to complete the hexagon by setting off the radius, six times around the circumference, joining the points so found.

Principle.—The side of a hexagon is equal to the radius of its circumscribing circle.

Second Method.—1st. At B and A, construct lines, making angles of 60° with AB produced in each direction (see PAB and PBA), and make each of these lines equal to AB.

2d. Join CF, and at each of these points draw lines equal to AB, and making angles of 60° with CF.

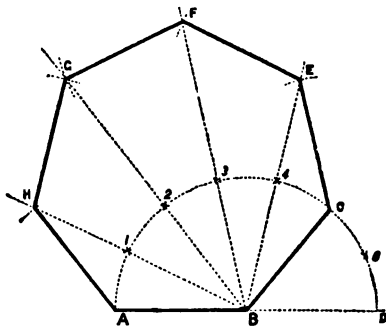
3d. Join DE, which will complete the hexagon.

Principle.—A regular hexagon has six equal sides, and six angles of 120° each.

PROB. 41. To construct any regular Polygon on a given side.

Let AB be the given side (Fig. 71), and let the figure taken to illustrate the solution, be a regular heptagon.

FIG. 71.



1st. Describe a semicircle, with AB for its radius.

2d. Divide this semicircle, by trial, into seven equal parts.

3d. Draw Bc to the fifth point of division, from A.

4th. Through the intermediate points of division, draw indefinite lines, from B, the centre of the semicircle Acd.

5th. With AB as a constant radius, describe an arc from c as a centre cutting Be at e; from e, cutting Bf at f; from f, cutting Bg at g; from A, cutting Bh at h; and from h, cutting Bg again at

g —to verify the former point g , whose intersection with the arc from f was an unfavorable one.

6th. Join the points thus found, and a regular heptagon will be formed.

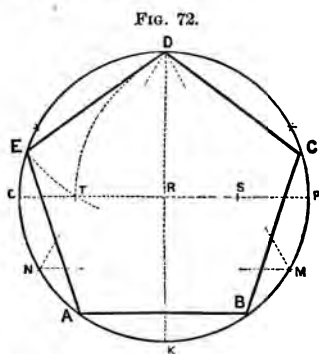
Principles.—1°. The diagonals of a regular polygon make equal angles with each other, and with the sides adjacent to the vertex from which they are drawn.

2°. The sum of the interior angles of a polygon equals $\alpha - 2$ times two right angles, where α equals the number of sides of the polygon.

3°. In a heptagon, one of its equal angles is therefore equal to $\frac{1}{7}$ of a right angle, or $\frac{1}{7}$ of two right angles; and is divided into five equal parts by the diagonals from its vertex.

PROB. 42. To inscribe a regular Pentagon in a given circle.

Let ACD be the given circle. (Fig. 72.)



1st. In the given circle draw the diameter, op , and the radius, rD , at right angles to each other.

2d. Bisect either radius, as rp , at s .

3d. With s as a centre, and sD for a radius, describe an arc cutting op at t .

4th. With D as a centre, and with Dt as a radius, describe an arc cutting the given circle at E .

5th. Take DE as a chord, and apply it five times to the circumference, and connect the points thus found, and the pentagon, $ABCDE$, will be completed.

Principles.—1°. sp being equal to sr ,

$$pt = st + sr$$

$$\text{and } rt = st - sr$$

$$\begin{aligned}\text{Therefore } pt \times rt &= st^2 - sr^2 \\ &= sD^2 - sr^2 = rD^2 = rp^2.\end{aligned}$$

Hence pt is divided medially, that is, in extreme and mean ratio, in \wedge .

2°. Again: $pt = rp + rt$

and $rp = ro$; and $rt = ro - ot$

$$\begin{aligned}\text{Hence } rp^2 &= ro^2 = pt \times rt = (ro + rt) \times rt = ro \times rt + rt^2 \\ &= ro(ro - ot) + rt^2 \\ &= ro^2 - ro \times ot + rt^2.\end{aligned}$$

(Cancelling ro^2 , and transposing, $rt^2 = ro \times ot$. Hence ro is divided medially at t , by this construction.

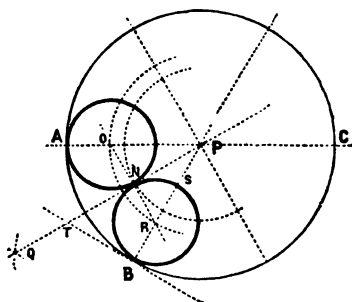
3°. The side of an inscribed decagon, is the larger segment, rt , of the radius thus divided.

4°. The side of a pentagon, $DE = Dt$ equals the square root of the sum of the squares of radius and the side of the inscribed decagon, $= \sqrt{Dr^2 + rt^2}$.

PROB. 43. *To describe any number of Equal Circles tangent to each other, exteriorly, and to a given circle, interiorly.*

Let ABC be the given circle. (Fig. 73.)

FIG. 73.



1st. Divide the circumference into the required number of equal parts; six, for example.

2d. Draw radii, PA , PB , etc., through the points of division.

3d. Bisect any one of the equal angles, as APB , made by these radii.

4th. At either of the points of division, adjacent to the bisecting line, draw a tangent, as Bt , and note the point t , in which it meets the bisecting line.

t , make tn equal to tB .

6th. At n , draw no and nr , perpendicular to Pt .

7th. At the points o and r , draw circles with the radii on and rn , and they will be two of the required circles. The centres of the four remaining circles will be, where a circle with Po as a radius, crosses the remaining radii, as PC , through the points of division, and their points of tangency will all be on a circle with its centre at P , and with Pn for a radius.

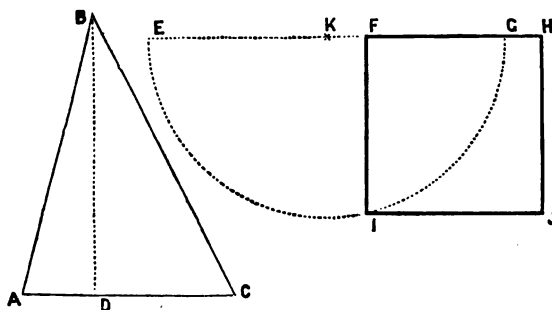
Principles.—1°. In a quadrilateral— $tnrB$ —if two adjacent sides— Bt and tn —and the angles contained between them and the remaining sides, are equal, those remaining sides, nr , and Br , will be equal.

2°. All radii of any one circle are equal.

3°. Any point in the bisecting line of an angle is equidistant from the sides of that angle.

PROB. 44. *To construct a Square, equivalent to a given triangle.*
Let ABC be the given triangle. (Fig. 74.)

FIG. 74.



1st. Taking any side AC as a base, construct BD , the altitude, perpendicular to AC .

2d. By Prob. 28, find FI , a mean proportional between the base and semi altitude of the triangle.

3d. On FI , complete the square $FIJH$, which will be the required square.

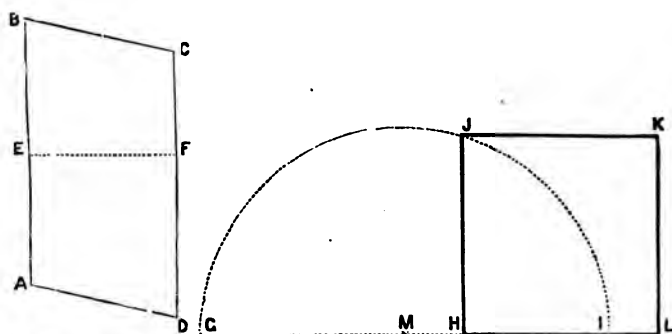
Principles.—See Probs. 28 and 38.

PROB. 45. *To construct a Square, equivalent to a given parallelogram.*

Let $ABCD$ be the given parallelogram. (Fig. 75.)

1st. Construct any perpendicular, as EF , between the bases, AB and CD , of the parallelogram.

FIG. 75.



2d. By Prob. 28, construct HJ , a mean proportional between the base and altitude of the given parallelogram; making HG equal to AB , HI equal to EF , and drawing a semicircle on GI .

3d. On HJ , construct a square, $HJKL$, which will be the required square.

Principles.—1°. See Prob. 28.

2°. The area of a parallelogram equals the product of its base by its altitude; the area of a square equals the product of one of its sides by itself.

Remarks.—(a) Were the given figure a rectangle, it is plain that the side of the equivalent square would be a mean proportional between two of its adjacent sides.

(b) Any polygon may be transformed into an equivalent square by reducing it to an equivalent triangle, which may then be changed into an equivalent square by Prob. 44.

Remark.—Careful attention to the first thirty-two of the preceding problems will enable the student to solve the following, and many others, of which the last thirteen are specimens.

PROB. 46. Draw a circle, tangent to a straight line and to a circle, at a given point on the circle.

PROB. 47. Draw a circle tangent to two straight lines and with its centre on a given straight line.

PROB. 48. Construct an isosceles triangle which shall be equivalent to a given scalene triangle having the same base.

PROB. 49. Construct a triangle equivalent to any given irregular polygon.

PROB. 50. Upon a given line construct a rectangle equivalent to a given rectangle.

CHAPTER IV.

GENERAL AND PRACTICAL PROBLEMS OF COMPOUND CURVES, COMPOSED OF CIRCULAR ARCS.

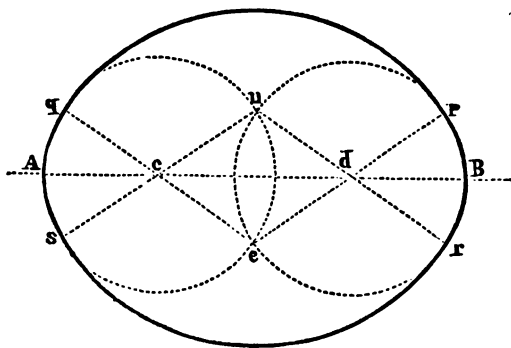
221. A compound curve is one which is composed of two or more simple arcs, tangent to each other.

All that is necessary, in order that two circular arcs shall be tangent to each other, is, that their centres should be in a straight line passing through the point of tangency.

An oval is an oblong curve, returning into itself, and composed of arcs of circles tangent to each other.

PROB. 1. *To describe an Oval, composed of four circular arcs.*
Fig. 76.

FIG. 76.



1st. Draw any straight line, AB, of indefinite length.

2d. Take any two points, *c* and *d*, on this line, as centres; and with any radius, greater than half of *cd*, describe two circles, intersecting each other at *n* and *e*.

3d. Through *n* or *e*, or both, draw lines, as *ep* and *eq*. These lines determine the points of tangency of the arcs, *rs* and *pq*, with the arcs drawn with *c* and *d* as centres.

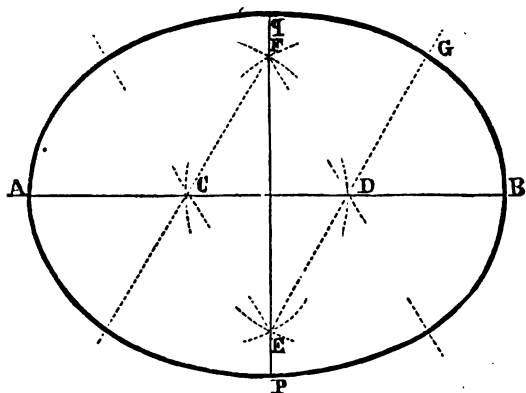
4th. Draw arcs, *rs* and *pq*, with *ep* or *eq* as a radius, and with

e and n as centres. These, with the arcs pr and qs , will complete the required oval.

Principles.—(See the remarks immediately preceding this problem.)

PROB. 2. *To draw an Oval, having a given longer axis.* Fig. 77

FIG. 77.



Let AB be the given axis.

1st. Divide AB into three equal parts, at C and D .

2d. Proceed as in the last problem, to take C and D as centres for small arcs, and the intersections, E and F , as centres for large arcs tangent to these, and having radii equal to EG .

Principles.—(See the last problem.)

Remarks.—(a.) C and D may be taken anywhere on AB ; only so that DB and AC shall be equal, and each greater than half of CD , if E and F are to be definitely located. If the circles on C and D as centres, do not intersect, any lines through C or D will meet pq in the centre of an arc, tangent to these circles.

(b.) pq , at right angles to AB , at its middle point, is the shorter axis of the oval; and it appears that the parts of the oval on each side of each axis, are similar. Thus qBp , turned over about pq as an axis, would coincide with qAp . Such a curve is said to be *symmetrical*, with respect to its axes.

PROB. 3. *To construct on a given line a Semi-Oval, composed of three arcs, two of which are to have equal radii, and passing through a given point on the bisecting perpendicular of the given line.* Fig. 78.

Let AB be the given line, and D the given point on the bisecting perpendicular; and let CD be less than CA.

1st. From A and B, lay off AE and BQ, each less than CD.

2d. Make DF equal to AE or BQ.

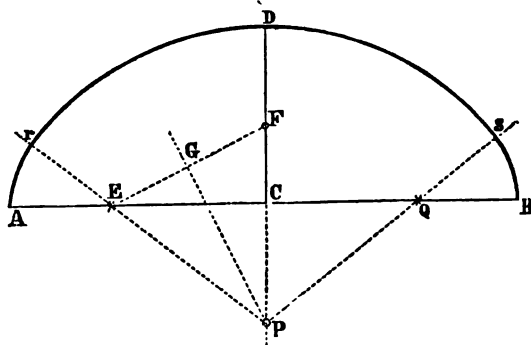
3d. Join E and F.

4th. Bisect EF, and note the point P, where the bisecting perpendicular meets DC produced.

5th. Draw the indefinite lines, Pr and Ps. Then E and Q will be the centres of the arcs Ar and Bs, and P the centre of the arc rDs. Therefore draw these arcs accordingly.

Principles.—1°. (See remark (b), Prob. 2.)

FIG. 78.



2°. A line, EF, which joins two points equally distant from the extremities, *r* and D, of two radii of an arc, is parallel to the chord of that arc.

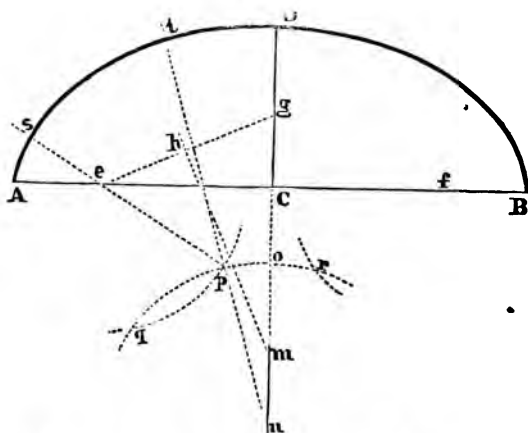
3°. The perpendicular to the middle point of a chord of an arc, passes through the centre of that arc.

Remark.—To make a well proportioned oval by this method, CD should not vary much from two-thirds of AC; and AE should be but little less than half of AC.

PROB. 4. *Having given a line, and its bisecting perpendicular, and two other lines, intersecting at a given point of the perpendicular, and making equal angles with it, it is required to draw a curve, having four arcs; two of them to be tangent to the given intersecting lines at the given point; two to have their centres in the first given line, and those on the same side of the bisecting perpendicular to be tangent to each other.* Fig. 79.

3d. Find m , where the bisecting perpendicular of eg meets DC produced

FIG. 80.



4th. [*Dm* would be the required radius, if there were to be out three centres. It will be evident, on trial, if not otherwise, that if there is to be an intermediate radius, not having its centre on either of the given lines, this radius must be less than *Dm*, and that the remaining radius must be longer than *Dm*.] Hence, take *n* and *o* at pleasure, making *Dn* greater than *Dm*, and *Do* less than *Dm*.

5th. [Suppose the figure complete; we may view it in this way. Imagine that as the radius nD sweeps over the paper, turning about the point n as a centre, it will, at some moment, become immediately changed to the intermediate length, pt , its *outermost* point t not being changed. Also suppose, that as the radius eA sweeps around the point e , it will, at some moment, change to the length ps , equal to pt .] Accordingly, describe an arc op , about n as a centre, on being the difference between the largest and the intermediate radius; and then, about e , describe an arc, whose radius is equal to go , the difference between the intermediate radius pt , and the least radius Ae . These arcs intersect at p , giving the centre for one of the intermediate radii.

6th. Produce the arc po to r , and make or equal to po ; then r will be the centre for the other intermediate radius. r may also be found by intersecting or by an arc, from f as a centre and having a radius equal to ep .

7th. Draw the indefinite lines, ps and npt , and similar ones through r and f .

8th. Then, e is the centre of the arc between eA and ps ; p is the centre of the arc between ps and npt ; and n is the centre of the arc between npt and the line through n and r produced, and passing through D . Let these arcs be thus drawn, and also the arcs having r and f as centres.

Principles.—The principles of the previous problems, with the remarks above enclosed in brackets, explain this construction.

Remarks.—(a) The arcs having e and n as centres, intersect at two points, p and q ; p , only, is used, as is evident on inspection.

(b) A knowledge of the higher branches of mathematics enables one to construct this figure and figure 3 in various other ways, some of them having particular given ratios existing between the radii. A very simple special case is given in the next problem.

(c) These oval curves are familiarly called "basket handles" by the French, on account of their shape. They are much used in the arches of stone bridges and tunnels. The pointed curve (Fig. 4) forms the outline of a depressed Gothic arch, as formed of circular arcs.

PROB. 6. *In the last problem, let both the extreme centres and radii be given, and the point of tangency of the middle arc and the arc having the longest radius, and let it be required to find the centre of the middle arc.* Fig. 81.

Let O and e be the given centres, on the axes AC and AB , OC and eB the given radii, and D the given point of tangency.

1st. Draw DO , and on it lay off Dg , equal to eB , the radius of the smallest arc.

2d. Draw ge , bisect it, and draw the bisecting perpendicular, hp .

3d. From p , where hp meets DO , describe the required intermediate arc, DF .

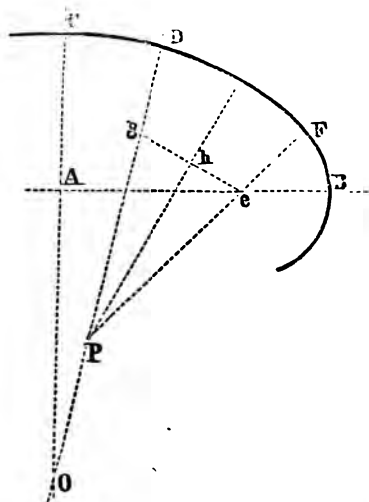
Principles.—See (221), and Prins. 2° and 3° of Prob. (3.) Also Prob. (4.)

222. An ellipse is an oval curve formed by cutting across a cone obliquely. The artificial ellipses, or ovals composed of circular arcs, are most pleasing when they most closely resemble a true ellipse. According to the properties of the ellipse, which the student will find in the study of the Conic Sections, this resemblance will be most fully secured by making the *greatest* radius, as OC ,

Fig. 81, a third proportional to CA and AB; and the *least* radius, Be, a third proportional to AB and AC. That is, we should have:

$$\begin{aligned} CA : AB &:: AB : CO, \text{ and} \\ AB : CA &:: CA : Be. \end{aligned}$$

FIG. 81.



A mean proportional between CO and Be may then be assumed to be the best value for the intermediate radius, Dp.

PROB. 7. To draw a Curve, tangent to two parallel lines, at a given point on each line, and composed of two arcs tangent to each other. Fig. 82.

Let AD and BC be the given lines, and A and B the given points.

1st. At A, draw the indefinite line Ao, perpendicular to AD.

2d. At B, draw BD, perpendicular to BC.

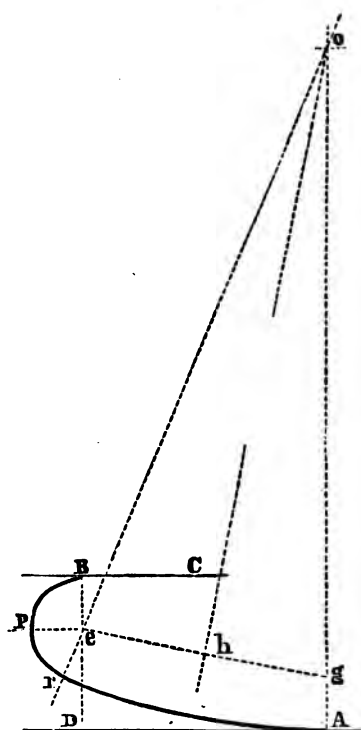
3d. Make Be equal to any distance, less than half of BD.

4th. Draw ep parallel to BC, make it equal to eB, and by Prob. (3) construct the quarter oval prA.

5th. With the centre, e, produce the arc, rp to B, which will complete the figure.

Principles.—See Prob. (3.)

FIG. 82.



PROB. 8. *Having the same given quantities as in the last problem, let the required Curve be composed of three arcs tangent to each other.* Fig. 83.

Let AD and BC be the given lines, and let A and B be the given points.

1st. Draw the indefinite line, AP, perpendicular to AD, at A.

2d. At B, draw BD perpendicular to BC.

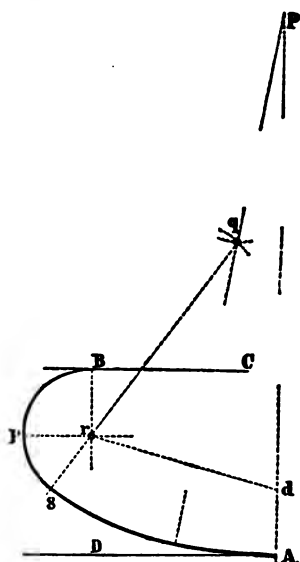
3d. Make Br equal to anything less than half of BD—really, about one-third.

4th. Make rp parallel to BC, and equal to rB ; draw rd — Aa being equal to rB —and by Prob. (5) or (6), construct the quarter oval of five centres, psA .

5th. With r as a centre, produce the arc sp to B, and the problem will be solved.

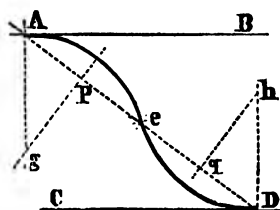
Principles.—See Probs. (3) and (5), or (6).

FIG. 83.



PROB. 9. *Having given two parallels, to draw a Reversed Curve tangent to each parallel, at a given point, and composed of two arcs tangent to each other.* Fig. 84.

FIG. 84.



Let AB and CD be the given parallels, and A and D the given points.

1st. Join A and D.

2d. On AD, assume any point *e*, at which the arcs shall be tangent to each other.

3d. At A and D, draw the perpendiculars, Ag and Dh.

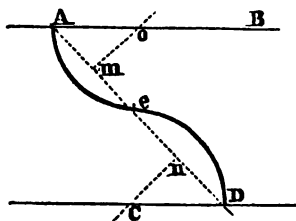
4th. At *p* and *q*, the middle points of Ae and eD, draw the perpendiculars, pg and qh.

5th. With g and h as centres, and gA and hD as radii, describe the arcs Ae and eD , which will form the required curve.

Principles.—1°. When two equal angles— gep and qeh —having a common vertex— e —have a side of each— pe and eq —in the same straight line, and the remaining sides— ge and eh —on opposite sides of this line, those other sides will form one straight line— geh
2°. See (221).

PROB. 10. *Having given two parallels, it is required to draw a Curve composed of two tangent arcs, each arc having its centre on one of the parallels, and passing through a given point on that parallel.* Fig. 85.

FIG. 85.



Let AB and CD be the given parallels, and A and D the given points.

1st. Join A and D .

2d. Mark any point, e , on AD , at which the two arcs shall be tangent.

3d. At m and n , the middle points of Ae and eD , draw the perpendiculars, mo and nC ; and note the points o and C , where they meet the given parallels.

4th. With o and C as centres, describe arcs, passing through A and D , and they will be tangent at e , and will compose the required curve.

Principles.—See the last Problem.

Remarks.—(a.) The last four problems are general solutions of the problems used in the construction of compound "Architectural mouldings." The curves of Figs. 82 and 83 form the outlines of the "moulding" called a "scotia," which is one of the "members" of the "base" of a "column." The curves of Figs. 84—"Cyma Recta"—and 85—"Cyma Reversa,"—form the outlines of some of the "mouldings" of the "cornices" of buildings. Other simple "mouldings," whose outlines are composed of single

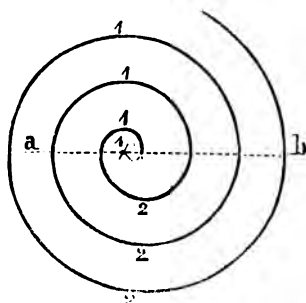
Principle.—If the equal distances between two lines are not parallel, the lines cannot be parallel.

Remarks.—(a.) This is a general solution of the problem: to determine the swell—"entasis"—in the outlines of the columns of edifices. *Bp* represents the vertical line joining the centres of the bases of the column, and *nc* represents the outline of one side of the column.

(b.) *nc*, as a curve, is a portion of the curve called a conchoid.

PROB. 12. To construct a Spiral, whose radii shall be in arithmetical progression. Fig. 87.

FIG. 87.



1st. On any line, *ab*, mark two points, 1 and 2.

2d. With 1 as a centre, describe the semi-circles marked 1; having their respective radii equal to the distance 1—2, three times 1—2, five times 1—2, etc.

3d. With 2 as a centre, describe the arcs marked 2, having their radii equal to twice 1—2, four times 1—2, six times 1—2, etc., and tangent to the arcs marked 1.

Principles.—See (221).

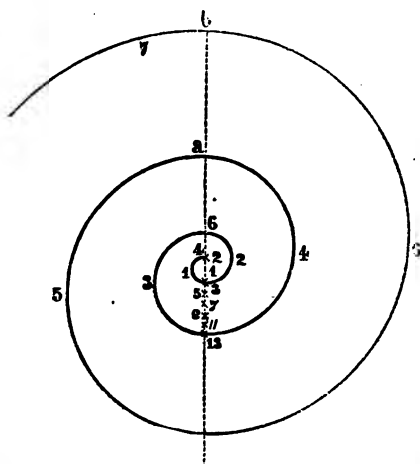
Remarks.—(a.) A spiral, as the figures 53 and 54 show, is a curve, which differs from a circle in having its circumference continually becoming further and further from the centre, which centre is called the "pole," in speaking of a spiral.

(b.) One complete turn of the spiral, as from *a* around to *b*, Fig. 88, on the same straight line, passing through the pole, 1, is called a spire.

(c.) Real spirals are not composed of tangent arcs. The figures are therefore only imitations, sufficiently accurate, however, for many practical purposes.

PROB. 13. To draw a *Spiral*, whose radii shall be in geometrical progression. Fig. 88.

FIG. 88.



1st. At any point, as 1, on either side of any straight line *ab*, draw a semicircle with any small radius, 1—2.

2d. If the given ratio of the radii be *two*, let each successive semicircle begin where the preceding one ended, and let its radius equal the diameter of the one just preceding. Thus 2 is the centre of the arc 2, the point 3 that of the arc 3. Or—

3d. If the given ratio be $\frac{3}{2}$, for example, and the first radius = 2, for example, the second radius would be $\frac{3}{2} \times 2 = 3$; the third = $\frac{3}{2} \times 3 = 4\frac{1}{2}$; etc.

Principles.—See (221), and the remarks after the last problem.

Arch Fronts.

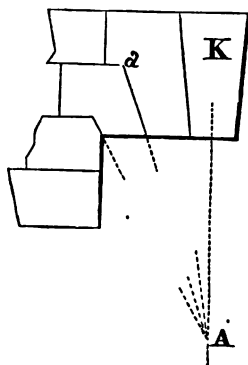
223. Pleasing and useful practical applications of the preceding problems of compound curves, are found in the designing of arch fronts.

Such designs also afford good elementary exercises in contriving the various pleasing and practicable arrangements of the stones forming both the immediate outline of the arch and its superincumbent wall.

224. The following definitions will facilitate the subsequent

descriptions of Figs. 89—93. These definitions relate, successively, to some of the *Masses*, *Surfaces*, *Lines*, and *Points* of the Arch.

Fig. 89.



225. An Arch is a passage usually of cylindrical form. If flat, as in Fig. 89, it is a Horizontal Arch, or *Plate Band*. If it has a semi-circular end, as in Fig. 91, it is a *full centred* arch. When the end is less than a semi-circle, the arch is called *segmental*. When the end is oval, as in Figs. 92, 93, a *Depressed Arch* is formed. These arches are called *Elliptical*, or *Polycentral*, according as their ends are true ellipses or compound curves. The wall sustained by the arch is called the *Spandril* or *Backing*, ABC, Fig. 92.

Single stones of the arch are called *voussoirs*. The highest one, as *b*, Fig. 91, is called the *Keystone*, but is no more essential to the stability of the arch than any other *voussoir* is.

The supports of the arch are called its *Abutments*.

226. The end, or front of the arch, as seen in the figures, is its *Face*. The inner, or under surface, is the *Intrados*. The outer, or top surface, is the *Extrados*. The level, as *ac*, Fig. 91, from which the arch rises, is called the *Springing Plane*, and in a segmental arch, the *Skew-back*.

227. The curved boundaries of the intrados and the extrados, are called *Face Lines*. The lines between the *voussoirs* are *Face Joints*. The width, as *ac*, Fig. 91, of an arch, is its *Span*. The height, as *mn*, Fig. 93, is the *Rise*. The straight lines which limit the intrados and extrados, as at *c* and *d*, Fig. 91, are the springing lines.

228. The highest point, as *n*, Fig. 93, in the intrados or extra

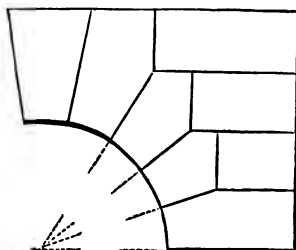
dos is its *crown*; the extremities of the face line are its *springing* points; and the centres of an oval face line are the *arch-centres*.

229. Passing now to explain the drawing of arches; we observe that, in most of the figures, only a half arch is shown, since the two halves are supposed to be alike in all of the examples; but the student is recommended to construct a whole arch in every case, also a segmental arch, as a good exercise in the exact drawing of symmetrical figures. A larger scale than is here shown, may also be advantageous, at least at first.

Fig. 89 represents a plate-band. Here the joints radiate to some point, as A, taken on the vertical centre-line of the keystone, K. The line KA is also the centre-line of the door or window covered by the plate-band. The overlapping at *a* is merely fanciful, and may involve the breaking off of the overlapping portion of the arch stone. There might have been a vertical joint at *a*, or one in Aa produced.

Fig. 90 is a semi-circular arch, so designed as to accord with

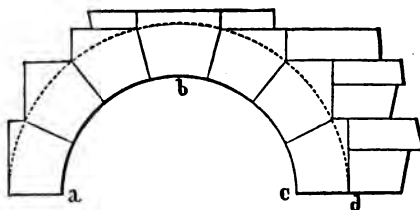
FIG. 90.



stones of equal thickness in the spandril. By locating the vertical joints differently, the voussoirs may be made to lap at pleasure over the horizontal stones.

Fig. 91 is a semi-circular arch, in which the radial joints are uni-

FIG. 91.

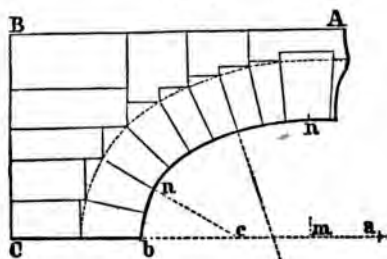


formly limited by a semi-circle concentric with the intrados. The

voussoirs are then completed by horizontal and vertical lines from the extremities of the radial joints, while the stones of the spandril are of various thickness. Sometimes arch stones are cylindrical on the outer surface, as if those of Fig. 91 were really limited by the semi-circle through d .

Fig. 92 is a three-centred arch. At c is one centre. ab is equal

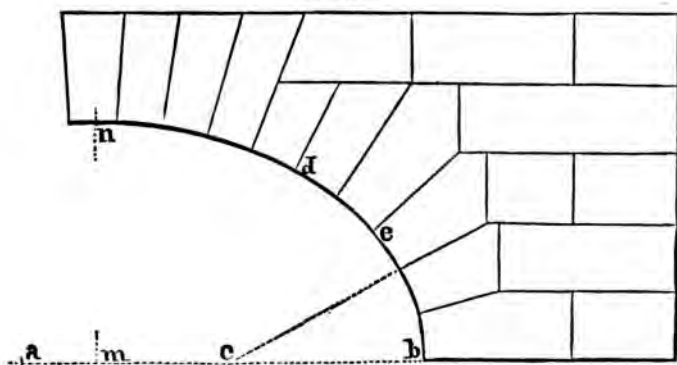
FIG. 92.



to the longer radius, which, if laid off from n , on nm produced, will give the middle centre, not shown in the figure. The line from the latter centre through c will limit the two arcs forming one-half of the arch. All the arch joints within the arc whose centre is c radiate from c . The other joints radiate from the centre on nm . All these joints are limited by a curve concentric with the face line, as in Fig. 91, and the spandril stones are fitted to them as shown in the figure.

Fig. 93 is also a three-centred arch, but designed to admit span-

FIG. 93.



dril stones of equal thickness. cb is the lesser radius, and ab the

greater, which, being laid off on nm produced, will give the middle centre. The first four voussoirs from b are thicker than the others, to accommodate the equal spandril stones.

Remarks. (a.) In all polycentral curves an unsightly flatness is observable in the region de of the junction of the two arcs. This is owing to the sudden and harsh change of radius from the length ab to the length cb . In the true ellipse there is a continual imperceptible change in the rate of curvature from b to n , producing a curve of delicate beauty, illustrative, in comparison with the polycentral curve, of the striking contrast in many cases between the perfection of nature and the imperfection of art.

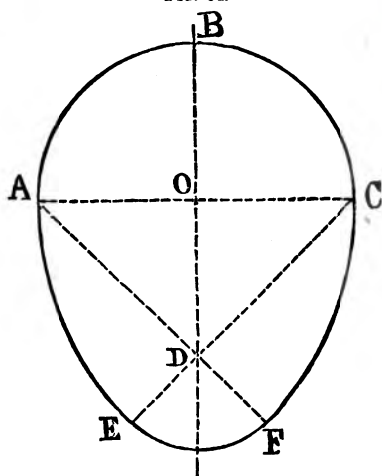
(b.) By the aid of the examples now given, the student can proceed to exercise himself in making other designs of similar character, including five-centred arches.

Ovals, or Egg-Forms.

230. We close this chapter with a brief notice of these, the most beautiful of geometrical decorative curves.

It must be premised that a *natural curve* is one which, like a circle, or an ellipse, results from one uniform law of construction at all of its points; as when a circle is formed according to the law

FIG. 94.



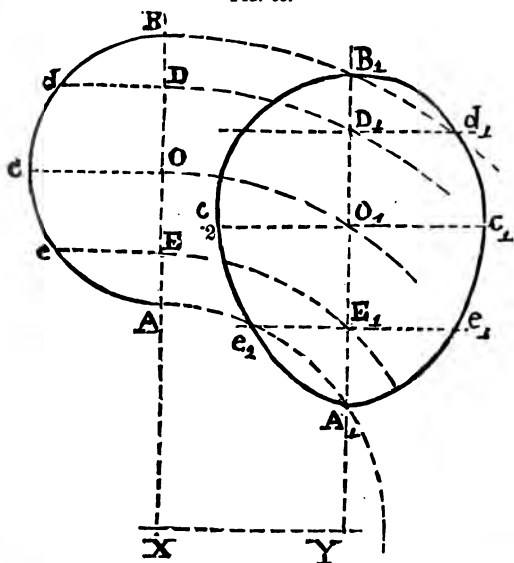
of one uniform distance, in a plane, of all its points from one fixed point, the centre. An *artificial curve*, on the contrary, is com-

posed of a combination of arcs of some natural curve, as seen in the various preceding curves described in this chapter.

This being understood, egg-forms, or ovals, not being found in mathematical treatises, except as special cases of quite complicated curves, it results that the egg-forms commonly described in practical treatises, are the quite rudely artificial ones, drawn as in Fig. 94.

Here O is made the centre of the semicircle ABC , the extremities, A and C , of whose diameters are then made the centres of CF and AE . Finally, these arcs are limited by the radii through D , found by making $OD = OA$, and D is made the centre of the arc EF .

FIG. 95.

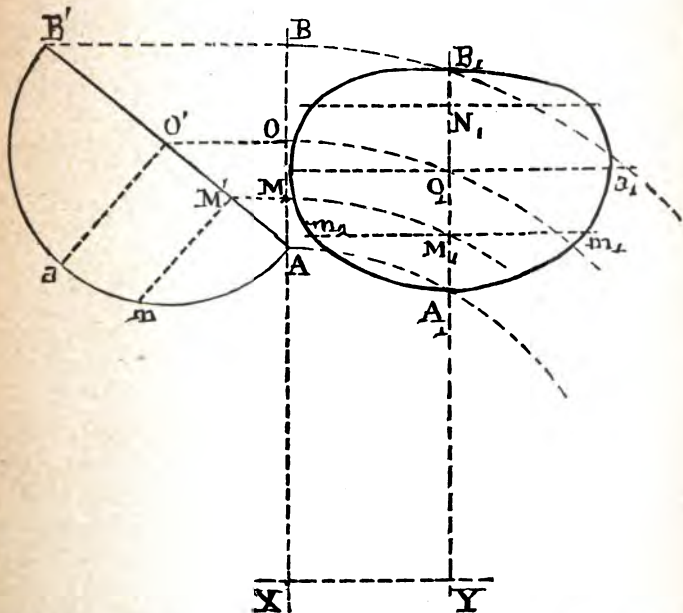


A somewhat better oval was devised by an English decorator, D. R. Hay, and styled by him a "*composite ellipse*," it being composed of arcs of ellipses, as described in his elementary work on "*Symmetric Beauty*;" but this curve is still artificial, according to the above definition, and thence inferior in grace to a truly natural curve.

Curves which are the sections of a surface cut by a plane, are natural curves, when the surface is formed by some uniform law. Thus the plane sections of a cylinder or a cone are natural curves.

Natural ovals, or egg-forms, can be conveniently formed in this manner in different ways. One of them is fully shown in my "*Elementary Free-Hand Drawing, Part III.*" Another simple method is as follows: On AB, in the paper, as a diameter, let there be a circle perpendicular to the paper; half of it shown, after turning it about AB into the paper, at AeB . Next let this circle (in its perpendicular position) be revolved about an axis, perpendicular to the paper at X. It will form a ring surface called an annular torus. If now this surface be cut by a plane, as YB, perpendicular to the paper, the section will be the oval $A_1e_1c_1B_1e_2$, which is thus constructed: Take $OE = OD$ for convenience, and draw Ee, Oc, Dd , perpendicular to AB. Describe the arcs EE_1, OO_1 ; and DD_1 with X as a centre. At E_1, O_1, D_1 , draw perpendiculars to A_1B_1 , as shown. On O_1c_1 make O_1c_1 and O_1c_2 each equal to Oc . Likewise lay off Ee on both sides of E_1 , as at E_1e_1, Dd as at D_1d_1 , etc. Connecting the points Ae_1c_1 , etc., we shall have the oval $A_1e_1c_1d_1B_1c_2$.

FIG. 96.



The form of this oval could be varied indefinitely in two ways: first, by varying the distance XY, keeping it, however, less than

XA; *second*, by varying the proportion between the radii, XA, and OA.

Another, and a more important variation consists in making a flattened oval, as in Fig. 96. This is done by simply placing the original circle in any vertical position oblique to AB, as on AB', and then projecting it in the direction B'B, at AB. This compresses it to an ellipse whose shorter axis is AB, and longer one equal to AB' and perpendicular to the paper at O. Proceeding now as before, to draw the arcs MM₁, etc., with X as a centre, make M₁m₁ = M₁m₂ = M'm: O₁a₁ = O'a, etc., and we find the flattened oval A₁m₁a₁B₁.

This oval can be varied as above described in Fig. 95; also by varying the angle BAB'.

Ovals, and other curves, not circular, being first constructed by points, as just shown, they are drawn by means of *irregular curves* (33). In doing this, choose a portion of the curved edge, which will connect as many points at once as possible, taking care that the successive portions are *truly tangent to each other*, and of a *regular gradation of curvature*.

The irregular curve, however, can never "supersede the necessity for the accurate and refined education of the eye, which is obtained by" the suitable previous study of free-hand drawing.*

* See PROJECTION, by E. A. Davidson, p. 20. Also my "ELEMENTARY FREE-HAND DRAWING."

DIVISION IV.

ELEMENTS OF TASTE IN GEOMETRICAL DRAWING.

231. Under this head it is proposed to present some of the principles of Art and of Taste, which are applicable to operations so conventional as are many of those of Geometrical Drawing.

These principles may be given as suggested by the following topics:

Properties of colors.	Borders.
Harmony of colors.	"Tone."
Lettering. { Captions.	Frame.
{ Title-pages.	
Special Ornaments.	{ Vignettes, Meridians.
	{ Symbolic devices, etc.

A.—Of the Properties of Colors.

232 White, or Solar light, may, as is well known, be separated into seven colors: viz. Red, Orange, Yellow, Green, Blue, Indigo, and Violet.

These distinctions are, however, less obvious than those between the three broader bands of the rainbow or spectrum, whose prevailing colors are Red, Yellow, and Blue.

From these three all other colors may be formed. Thus: Red and Yellow combined produce Orange; Blue and Yellow make Green; and Blue mixed with Red forms Purple. Hence the three colors—Red, Yellow, and Blue—are called *primaries*, and their compounds—Orange, Green, and Purple—are called *secondaries*. In like manner a mixture of

Orange and Green gives a color called *Citrine*.

Orange and Purple make *Russet*, and

Purple and Green make *Olive*.

These last three compounds are called *tertiary* colors; and their

compounds would be called *quaternaries*, but are in fact too obscure to have familiar specific names.

233. In the nomenclature of Chevreul, a celebrated French investigator of harmony of colors, and Professor to the Royal carpet and tapestry manufactory of the Gobelins at Paris, gradations between the secondaries and primaries are thus noted. Orange and Red is noted as Orange-Red, and Orange and Yellow is Orange-Yellow; Green and Blue is called Greenish-Blue, and so on.

Each of the three pure secondaries thus has an intermediate tint between it and the next primary, giving twelve colors in all. Finally, each of these twelve colors has five additional variations, formed by regular variations in the proportions of its two component colors, giving seventy-two colors in all.

234. This scheme can be well shown in a circular diagram of seventy-two equal radial bands, in which the primaries are 120° apart, the secondaries midway between them, and so on for the next gradations, the spaces between which are each divided into six equal parts by five radii, for the five variations of each specified color. The student should construct such a diagram.

235. It is next to be noticed, *first*, that all these clear colors, *i.e.* colors not mixed with either black or white, are called *hues* of color; *secondly*, that any *hue*, mixed with various proportions of black, forms a series of *shades* of that *hue*; *thirdly*, that any *hue* mixed with *white*, in various proportions, gives a series of *tints* of that color.

236. In the system of Chevreul the shades and tints of a *given hue* are called its *tones*, and he proposes twenty tones for each of the seventy-two "scales" in his scheme. Each *six scales* which are characterized by a prevailing color, as red, is called a "*hue*," and in *each scale* the *pure color* in its fullest intensity or depth is called its "normal tone." The other tones, radiating towards the centre, consist of an inferior intensity mingled with increasing quantities of white, while those which are further from the centre of the diagram are deepened by increasing quantities of black.

237. A reason may here be given for the superior beauty of shades and tints over that of hues. Pleasure is afforded to the mind partly in proportion to the partial reserve with which things agreeable are merely suggested to it, rather than, as it were, obtrusively set forth. For in this way a field is offered for the exercise of imagination. Thus the shades and tints suggest the hues.

But again, permanently ungratified curiosity to see these hues in their clearness would be painful, while yet that curiosity can be

gratified by the occasional display of these hues in small quantities. Hence, with the prevalence of shades and tints, points of brilliant hue decorate the objects of vision in the most acceptable manner.

238. *Shades and tints* are so often met with in the arts that they are often known by certain popular names, as follows :

Primaries and Black.—Red and Black produce "*Claret*;" Yellow and Black "*Drab*;" Blue and Black "*Blue-black*."

Primaries and White.—Red and White produce "*Pink*" and "*Flesh color*;" Yellow and White "*Straw color*;" Blue and White "*Pearl color*."

Secondaries and Black.—Orange and Black produce "*Brown*;" Purple and Black "*Dahlia*;" Green and Black "*Invisible Green*."

Secondaries and White.—Orange and White produce "*Buff*" and "*Cream color*;" Purple and White "*Lilach*;" Green and White "*Pale Green*."

It may be remarked here that certain *secondaries*, in which one of the primaries is largely in excess, receive specific names. Thus, red with a *very little* blue produces "*crimson*;" and red with a *very little* yellow produces "*scarlet*." So blue slightly inclining towards red is "*violet-blue*;" and again, "*pea-green*," or bluish green, is green *tinged* with blue—and in like manner we have *yellowish-green*.

239. *Grays.* Black and white, mixed in various proportions, give various "*tones*" of pure or "*normal*" gray. Any tone of gray, tinged with any other color, gives a gray named from that color. Thus, in cloud tints, and distant mountain shadows; also in wall tints, and in many fabrics, we have blue grays, purple grays, yellow grays, pink grays, etc.

Of Pigments.

240. Pigments are the material colors used in the arts, and this brief view of them will conclude this notice of colors. Pigments are divided into two groups, called "*Body colors*" and "*Transparent colors*." Body colors are those with which it is very easy to entirely cover up and hide any other color. Thus "*Lampblack*" and "*Vermilion*" are decidedly body colors, while cobalt blue and gamboge are transparent colors, and must be laid on very thick, and many times, in order to hide a color under them.

This distinction between colors being of comparatively small consequence to the geometrical draftsman, will only be alluded to in particular cases.

The colors ordinarily used by the draftsman are very few but

in fully colored drawings, as of locomotives, with partly pictorial surroundings, a number of colors are often used, both in thin tints and in thick coats.

BLUES.—*French Blue* is a very rich blue, much cheaper than Ultramarine, but nearly as brilliant, while it works better.

Smalt is a splendid violet color.

Cobalt is a very pure and quite strong sky blue, more clear and bright for drawing water lines, for example, than is Prussian blue.

Prussian Blue is a very deep, strong color, but tinged with green; and, since Indian ink is an intense "drab," the mixture of the two has quite a greenish cast, which may, however, be neutralized by a minute portion of carmine.

YEELLOWS.—*Gamboge*, a natural gum, which may be used in its unprepared state, is a bright yellow when used in pale washes, but is rather brown when laid on thick.

King's Yellow is a bright light yellow, having a slightly greenish tinge. There are also brilliant *Chrome* yellows.

REDS.—*Carmine* is a gorgeous crimson.

Crimson Lake is a little less vivid than carmine.

Vermilion, a compound of mercury, is a brilliant red, but so heavy that it does not flow readily, and when mixed in a tint with any other color its particles sink to the bottom, and even in the brush separate themselves from other tints, and so produce spottiness in the tint.

For copper, therefore, mix gamboge and carmine, rather than gamboge and vermilion.

Next to the preceding primaries are the following secondaries:

ORANGE is formed from Red and Yellow, as from Carmine or Crimson Lake, and Gamboge.

PURPLES of various shades are formed by mixing Prussian blue or Cobalt with Carmine.

GREENS.—*Emerald Green* is a brilliant light green. A very bright artificial Green may be made by mixing Cobalt and King's Yellow or Gamboge.

BROWNS.—*Burnt Sienna* is a very strong brown, nearly orange.

Sepia is a pale dull brown. It affords a fine color when mixed with a minute portion of Carmine, and either thus or in the pure state, is a fine substitute for Indian ink in the execution of drawings which are shaded only, without having the conventional tints added.

Burnt Umber is a beautifully clear dark-brown, very effective in tinted architectural drawings.

Bistre is a still darker brown, which affords an appropriate shadow color for Burnt Umber.

Yellow Ochre is a dark dull "buff," occasionally useful.

BLACKS.—Ivory-black and Lampblack are both good blacks, the former being a little more intense.

WHITES.—*Chinese White* is the purest and best working color "Constant White" is a chalky color, not satisfactory in its working

B.—Of Harmony of Color.

241. *Harmony of color* means agreeable combination of color, and is of two kinds, harmony by *analogy*, and by *contrast*.

Harmony of *analogy* exists between *slightly different tones* (236) of the same color, which will be near each other on the same radial band (234). Also between tones in *nearly related scales* of the same hue; such as would be found side by side at nearly or quite the same distance from the centre, and on consecutive radial bands.

Harmony of *contrast* exists between *tones* placed far apart on the same *scale*, or radial band; also between *scales* of the same hue, as far apart as possible, as between the reddest of the orange-reds and the yellowest of the orange-yellows; and finally, and most important, between *complementary colors*.

Complementary colors are *pairs of colors* which, together, contain all the primaries, in proportions to make white. They are especially agreeable, since white, or natural light, and hence its components taken in proportions to produce it, satisfies the eye. Each primary will thus be complementary to the sum of the other two; as *red*, complementary to *green*; *purple* to *yellow*; etc.

242. To enable the harmonious combinations of colors to be easily remembered, a subdivided hexagon may be prepared, with triangular appendages, as *agb*, as seen in the following diagram, in which the harmonious or "complementary" tints are arranged in a simple and perspicuous manner.

243. White, being the sum of all the colors, contains that in it which is in harmony with any given color, hence it is a familiar fact in matters of dress, etc., that white looks well with any color. Likewise black, being only negative, or the absence of all positive color, has nothing in it discordant with any given color; hence again it is equally well known that black looks well with any other color. Recollecting this, and also the formation of the *shades* and *tints*, it appears that *red*, *claret*, and *pink* are in harmony with *green*, *invisible green*, and *pale green*. See Fig. 97. *Yellow*, "*drab*," and *straw color* also harmonize with *purple*, *dahlia*, and *lilich*.

Blue, blue-black, and "pearl color," again, are in harmony with orange, brown, and "buff." In general, we conclude that any two colors found on the same axis, as *mk*, will harmonize.

For, put **R** = red ; **B** = blue ; and **Y** = yellow.

Also, **W** = white.

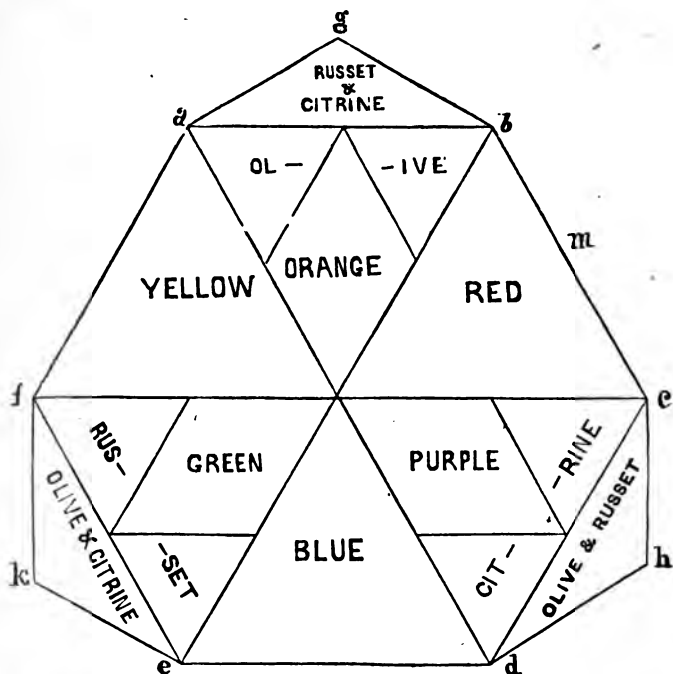
Then, for example :

Russet = orange + purple = $(R + Y) + (R + B) = 2R + Y + B$.

Green = blue + yellow = $Y + B$.

Russet + green = $2R + 2Y + 2B = W$., as required for complementary harmony of color.

FIG. 97.



244. Likewise, in the diagram of Chevreul (234), any two colors lying on the same diameter are "complementary" or harmonious.

If the above harmonies of complementary colors were alone agreeable, operations with the use of many colors would be greatly hindered. But various shades, tints, that is *tones* of the same color, associated in easy gradation, are pleasing also; and so are similar

tones, etc., of related colors, as of two orange "scales," one of which has more red in it than the other. Moreover, as we can see sharply in but one direction at a time, while we can hear clearly in all directions, analogy of color harmony, to harmony of sound, has a limit, and, in fact, it is only necessary that colors which are in immediate contact should be truly "complementary," that is, in harmony.

C.—*Lettering.*

245. The title to a drawing should answer distinctly the four questions—*What, Who, Where, and When*—*What*, including the use and scale; *Who*, both as to designer or inventor, and draftsman; *Where*, both as to the place, institution, or office where the drawing was made, and the locality of the object drawn; and *When*.

246. The use of lettering on a drawing having now been explained, the *design* and *execution* of the title may be treated under the following heads:

(a.) *Position*.—This should be below the middle of the sheet, if possible, as it is agreeable to see the space above the drawing left blank, as it usually is above the object itself, which is represented.

If the drawing is perfectly symmetrical, its title should have the same axis of symmetry as the drawing. If the drawing is unsymmetrical the title may be at either of the lower corners.

These principles do not apply to horizontal views, as maps of surveys, where the title may be wherever the shape of the plot affords the best place.

(b.) *Arrangement*.—An important element of beauty in a title is its outline. Its lines of letters should vary in length, so that the curve formed by joining their extremities should be a graceful one. Also the proportions of the title may appropriately be like those of the drawing. The *principal line* of the title, if made up of short words, may be extended, by separating the letters further, by the use of expanded letters, or by the addition of heavy dashes or other ornaments at the end. A *central point* in the title may well be secured by a *monogram*. If necessary, the title may well be mostly on a single horizontal line at the bottom of the sheet, the principal words being in the middle, and the subordinate ones at the two sides.

Moreover, horizontal lines should prevail in the direction of the lines of words in the title. Indeed, the title may be arranged wholly on horizontal lines with good effect, though an arched or

bow-shaped curve for the principal words may be adopted when the drawing includes some conspicuous arching lines.

(c.) *Size*.—The size of the title should be appropriate to that of the drawing. In particular, the rule has been proposed that the height of the largest letters in the title should not exceed three-hundredths of the shorter side of the border. Also, the relative size of the different portions of the title should correspond to their relative importance, the name of the object and its inventor being largest, and that of the draftsman, his location, and the date of his work being considerably smaller.

(d.) *Form*.—This may correspond in style and simplicity with the character of the drawing. Geometrical drawings are *most* appropriately lettered with geometrical letters, which, when neatly made, always look well. Any letters, however, having any kind of sharply defined and precise form, as German text, are not inappropriate to a geometrical drawing; but vaguely formed “rustic” or other free-hand letters are in bad taste on such drawings.

(e.) *Weight*.—Letters should correspond in conspicuousness or body of color with the rest of the drawing, not being obtrusive from great heaviness of solid black outline, or unobservable from excessive faintness. Also, violent contrasts of heaviness among neighboring portions of the title should be avoided; though there may be a gradual change, both of intensity and size, from the most to the least important words of the title.

(f.) *Finish*.—This should, first of all, not exceed in elaborateness the draftsman’s ability to execute it with perfect neatness and clearness. Then it should agree with the character of the drawing. Plain and simple letters look best on a similar drawing, while a complicated and highly finished drawing may receive letters of more ornamental character.

(g.) *Title-pages* to autographic collections of plates, being more independent of the plates than the title on a drawing, may admit more freedom of size and style of execution. Still, they should possess a certain conformity to the general character of the plates to which they are prefixed.

D.—*Borders.*

247. *Intelligence* is exercised in the *construction* of a drawing. *Fidelity* in its *execution*; and *Taste* in its *decoration*. All of these, indeed, together with *skill*, are exercised on each branch of the drawing, but each, especially, on its own appropriate department.

Provision is thus made for the manifestly complete action of the mind on the draftsman's work, and the border thus becomes the tribute of beauty to what it *incloses*, and thereby separates from external things as having a distinct claim to notice.

248. This, I think, is the whole general theory and significance of the border. Other minor principles of taste, which should regulate the border, are appended below.

(a.) For line-drawings the border should be a geometrical design, in lines, with curved or angular corners, or with combination of straight or curved lines, forming geometrical corner-pieces. These borders may vary in complexity from a rectangular border in single lines to borders which, though geometrical, may be elaborate and elegant, thus echoing the character of the drawings included by them. Thus: a plate of varieties of straight horizontal lines may have a plain rectangular border; one including oblique lines may include oblique lines in the border, either as a little tuft in each corner, a truncated corner, or a square set diagonally, etc. Plates embracing curve lines may have quarter-circle borders, either convex or concave inwards—of which the former have most decision. Such plates may also have little circles for corner-pieces. Borders may sometimes conform in a pleasing manner to the general outline of a drawing. Thus, an arched bridge may have a semi-oval upper border and a square-cornered border at the base of the drawing; and an ornamental device may crown the summit of the border.

(b.) When the drawing is a shaded one, containing, therefore, some free-hand work, the border may be partly free-hand also; but should still be largely geometrical in its design, and should represent a real border of substantial materials, corresponding to the subject of the drawing. Thus, the mouldings and ornaments should represent ornamental metallic castings, carvings in wood, mouldings in plaster, or scrolls and leaves of rolled metal; but garlands, tassels, and tendrils, etc., should not be introduced.

(c.) The border to a geometrical drawing should be like the drawing itself in being executed with the drawing-pen and brush, as well as with the mapping-pen. Free-hand pen-borders, representing the products of the soil, with cornucopias, little pen sketches of scenery, or similar agricultural or landscape devices, worked in as corner-pieces, are more appropriate on topographical drawings.

(d.) As to color, "primary" colors should not be largely introduced into the border; *first*, since they, when obtrusive, are

adapted to ruder or less impressible tastes than the "secondary" hues, shades, and tints (235) which are more gratifying to delicate tastes; and *secondly*, from the impertinent conspicuousness which they may give to the border.

Drawings which are shaded only in sepia or ink, or any dark neutral tint, may have the border done in the same, or in a dark complementary color. Tinted ink drawings are best finished with a plain ink border.

(e.) Perfect beauty embraces both decision and grace. These are united in *compactness*, joined with *easy curvatures*, as seen in a well-knit and agile frame. Hence a rambling or sprawling border is always vague, weak, and slovenly-looking.

(f.) Lastly, the draftsman should learn to keep his *ambition* and his *capacity* well balanced. That is, he should not attempt, on a well executed drawing, a border that he cannot worthily execute. Each one should be so reasonably contented with his known powers, as not to hinder their legitimate and successful exercise by unpromising attempts to exercise higher or different powers not yet, at least fully, possessed.

E.—"Tone."

249. This word is used figuratively by artists, it being borrowed from the art of music, and used to denote brightness—or "high tone" of color—as distinguished from dull or "low tones" of color.

250. The principle to be chiefly observed is, that there should be uniformity of tone throughout the drawing. Thus, very black shading should not be coupled with very faintly grained and tinted wood or stone-work, and very dark and black-grained stone-work should not be associated with a very faint border and title. The *absolute* darkness of the whole will generally be regulated by the constitutional preference of the draftsman.

251. According to the foregoing principles, the border to simple plates of line problems should be light as well as simple.

Plates of designs for pavements, at least when one design is made to fill a plate, are better without borders. A plate of light flat tints may have for a border *fine* double lines or a narrow strip of dark tint, in one or more shades.

252. Different draftsmen and writers on geometrical drawing recommend tints of different strength of color for the tinted parts of drawings. Faint tints are much more easily laid on, yet some desiring a correspondence between the depth of the color and the weight of the material, especially in case of metals, advocate deep

colors in tinted drawings, such as may be seen in many French lithographs of machinery.

On the other hand : First, as the draftsman's representations of objects are so artificial in *form*, that is, so unlike "pictures," they may consistently be equally artificial in color ; Second, the contrast of mild tints with white paper is as decided as that between machines, for example, and the blackened walls of the rooms in which they are often found, so that, considering also the superior evenness of light tints, they seem, on the whole, preferable.

F.—*The Frame.*

253. *The object of the Frame* is to afford a suitable protection to the finished drawing, and hence should be so subordinate in design and color as not to distract attention from the drawing.

254. The principles of the last topic, taken in connection with the fact that in geometrical drawings a large part of the paper is left blank, suggest that a gilt frame is, in general, preferable to a dark-colored wooden one. Occasionally the latter style of frame may be appropriate, as in case of a very darkly-shaded drawing on tinted paper, or of a drawing which very completely fills the paper.

It hardly need be said that a frame of plain mouldings is more appropriate for a geometrical drawing than is a carved or stucco-moulded frame. For ordinary geometrical drawings, nothing is prettier than a frame of butternut, or any light-colored native hard wood, enlivened by a gilt band.

G.—*Special Ornaments.*

255. These are : Vignettes, Meridians, and various symbolic and other devices, wrought into the title or corner-pieces. As examples of vignettes in a geometrical drawing, may be mentioned a miniature pictorial view of the object drawn, with its immediate surroundings ; or of the office, room, or institution at which the drawing was made. Similar ornaments on a topographical drawing, would be little pen-pictures of views seen on or from the premises surveyed. Emblematic devices on a geometrical drawing would not, perhaps, properly include anything but pictures of the drafting instruments, unless in case of title-pages to collections of plates, when, in a series of plates like those of this volume, the title-page might contain a picture of a pile of plates loosely scattered about. A volume of plates of shadows might have letters

casting shadows, or a group of small shaded figures in its title; one on machine drawing a little picture of a machine, etc.

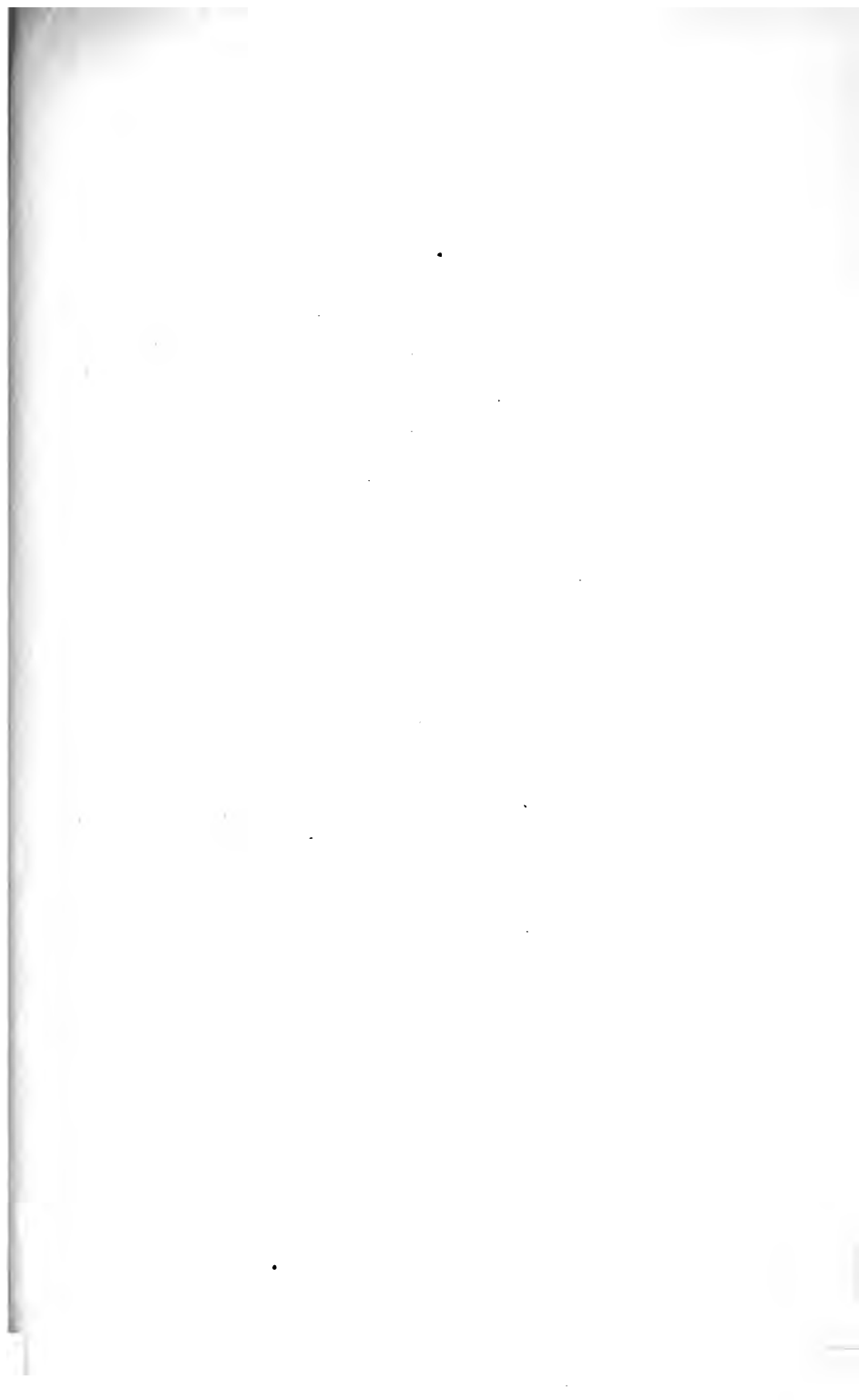
256. In respect to special ornaments, all straining after fanciful or capricious originality or effect should be avoided.

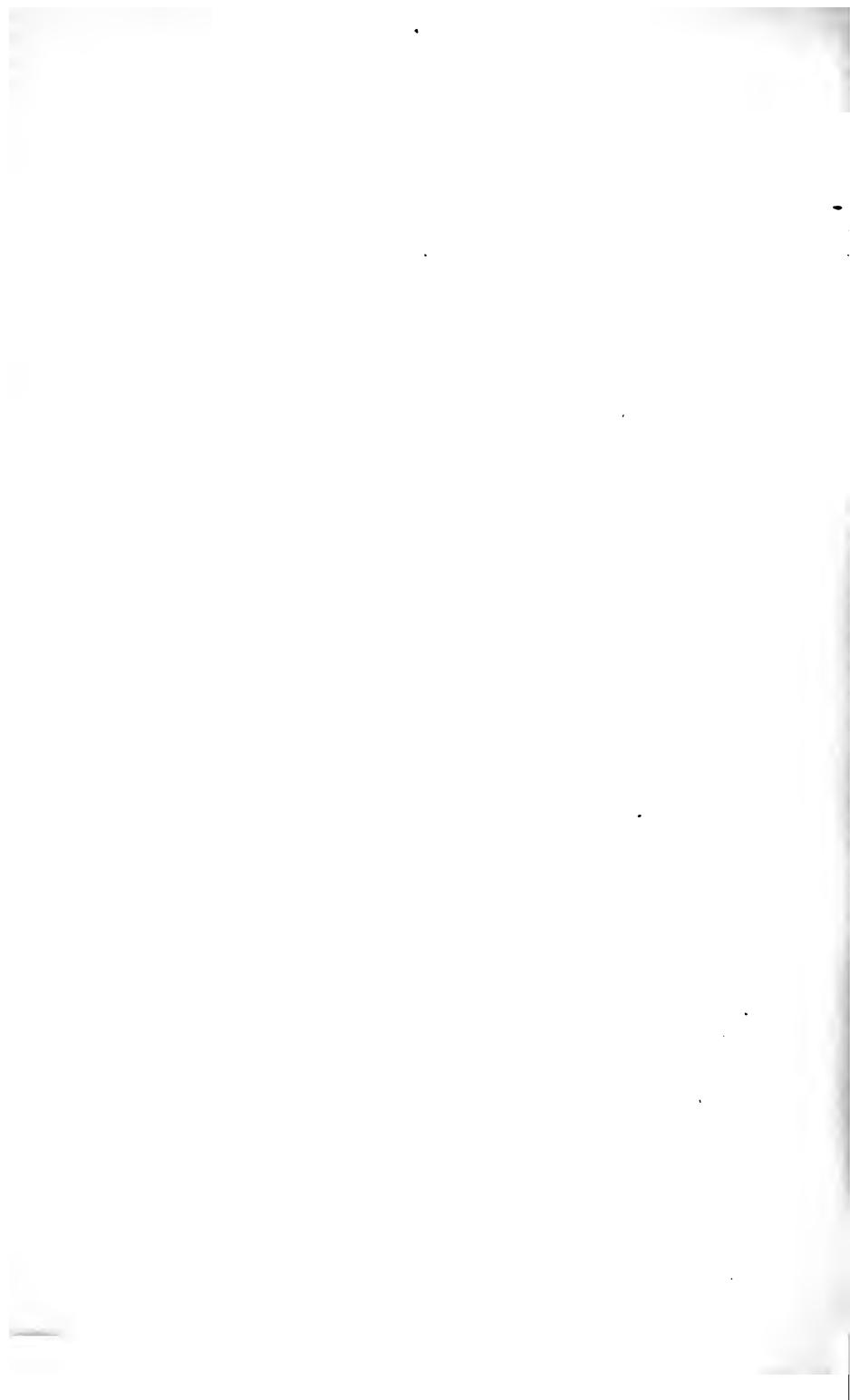
Forced, and hence unnatural, devices, or those which in any way are foreign to the character and associations of the drawing, are in bad taste.

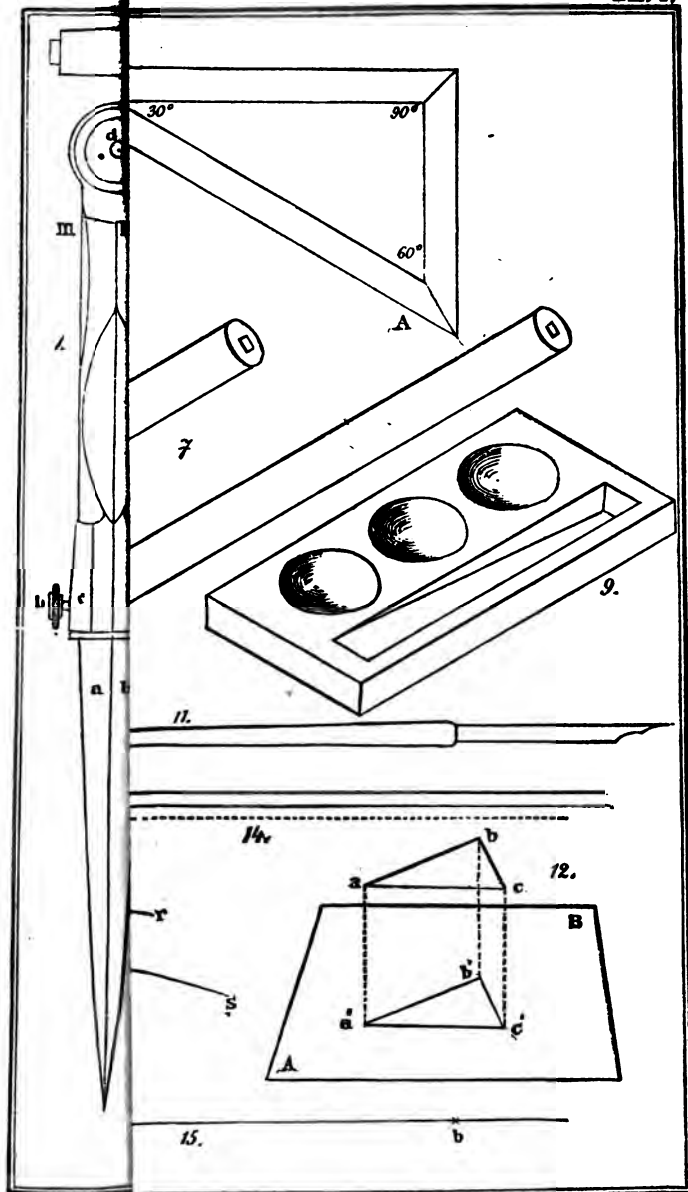
For each one, the best effect that he can produce is that which freely flows from the pleasant exercise of his own individual powers, without painful efforts at imitation or origination. Plain neatness is always agreeable where nothing higher can be afforded, but pretentious insignificance of device is always distasteful.

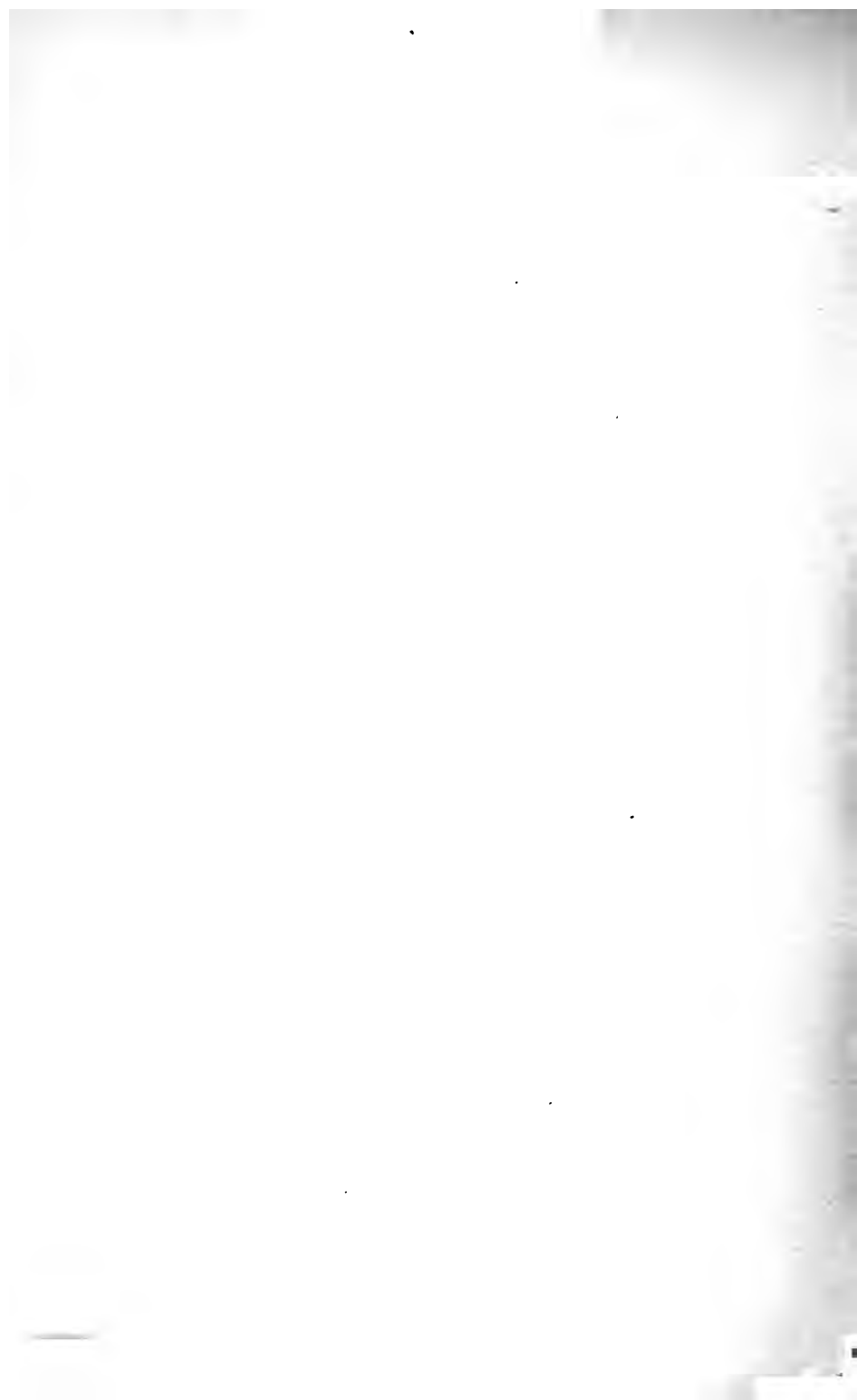
Finally, among the means of improving whatever original endowments of decorative taste appropriate to his art the draftsman may possess, the following are prominent, if not all-embracing: Familiarity with objects of beauty, and reflection upon their significance, with devotion to Purity and Truth.

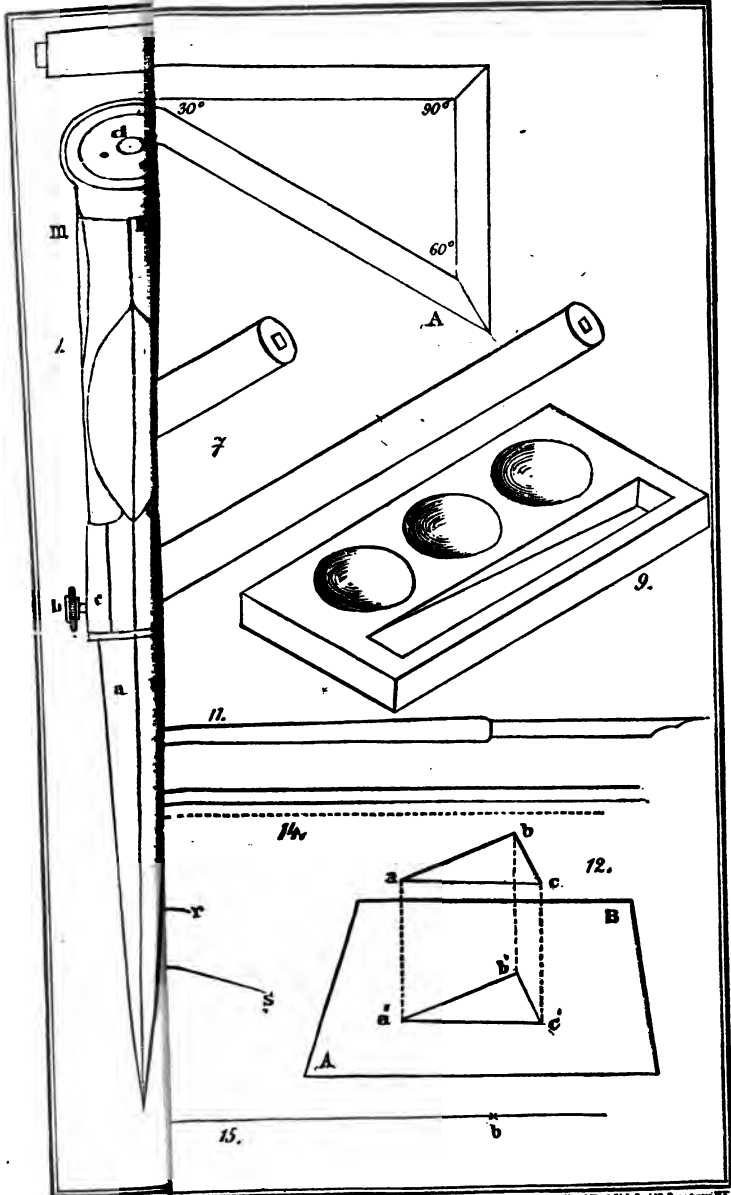
THE END.

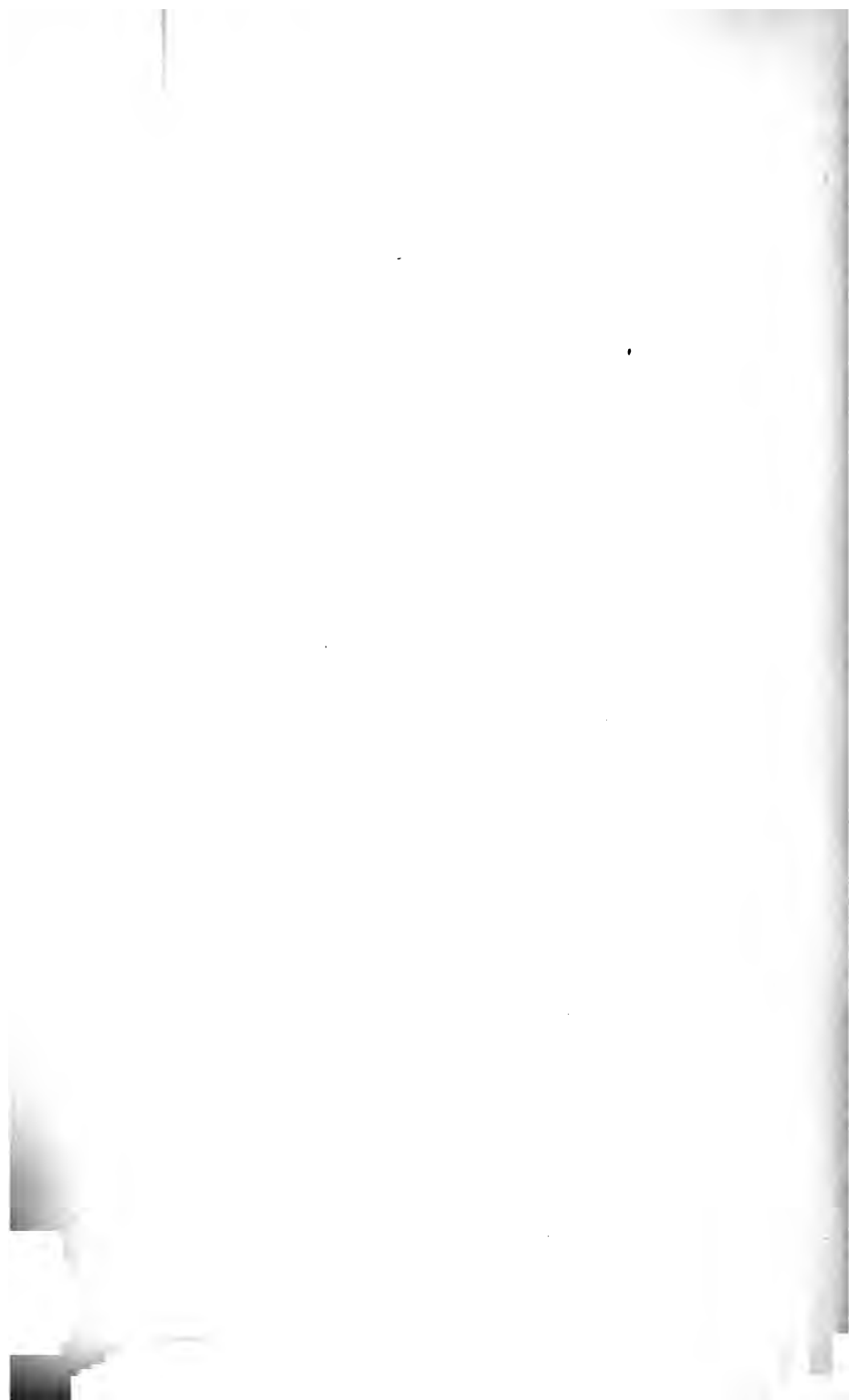


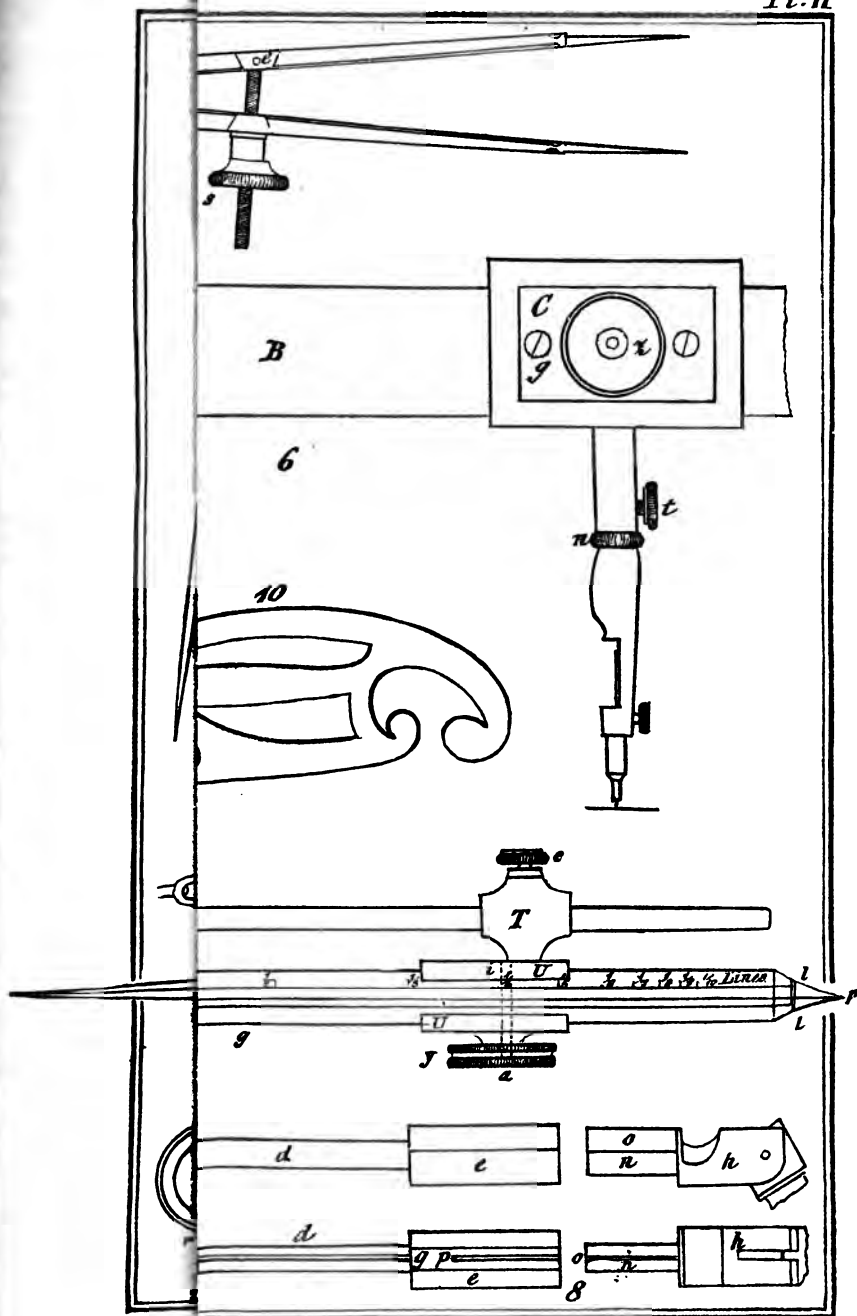














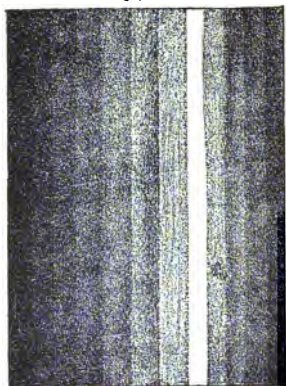
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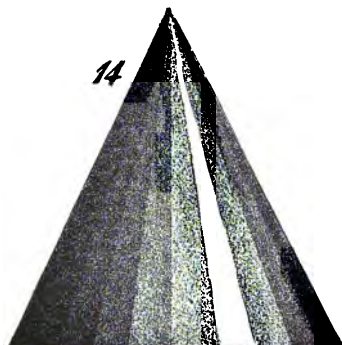


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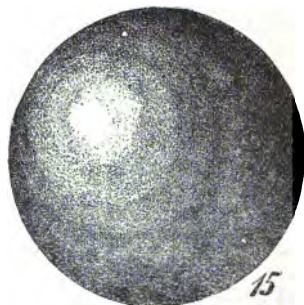
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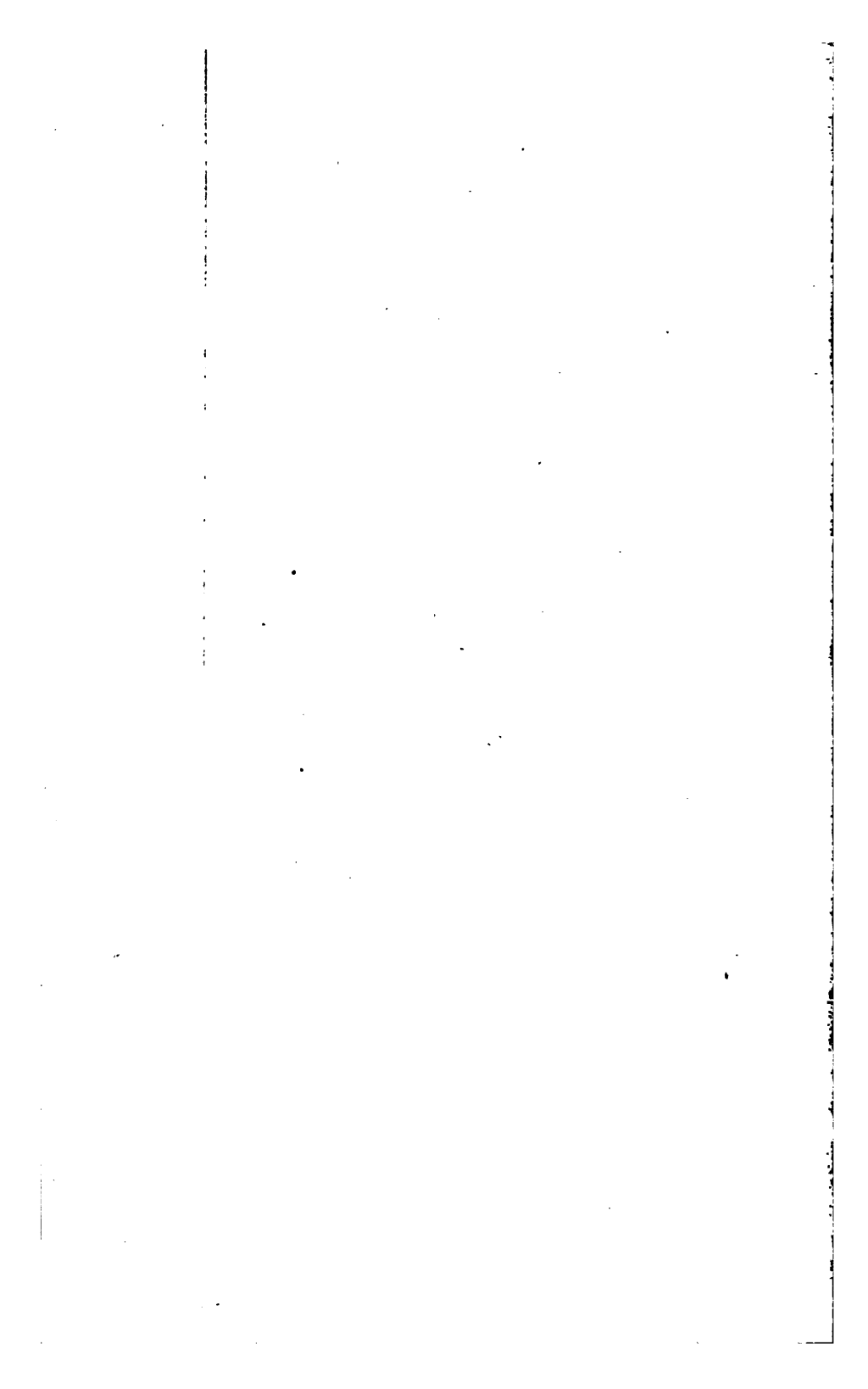


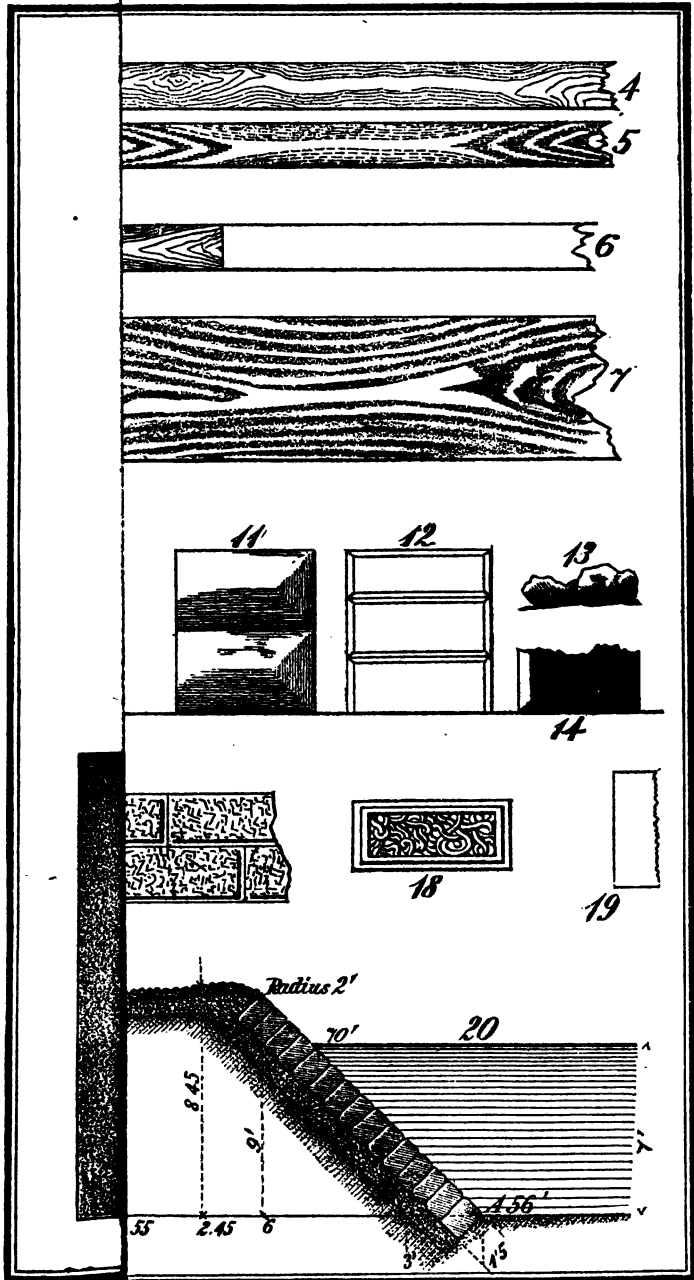
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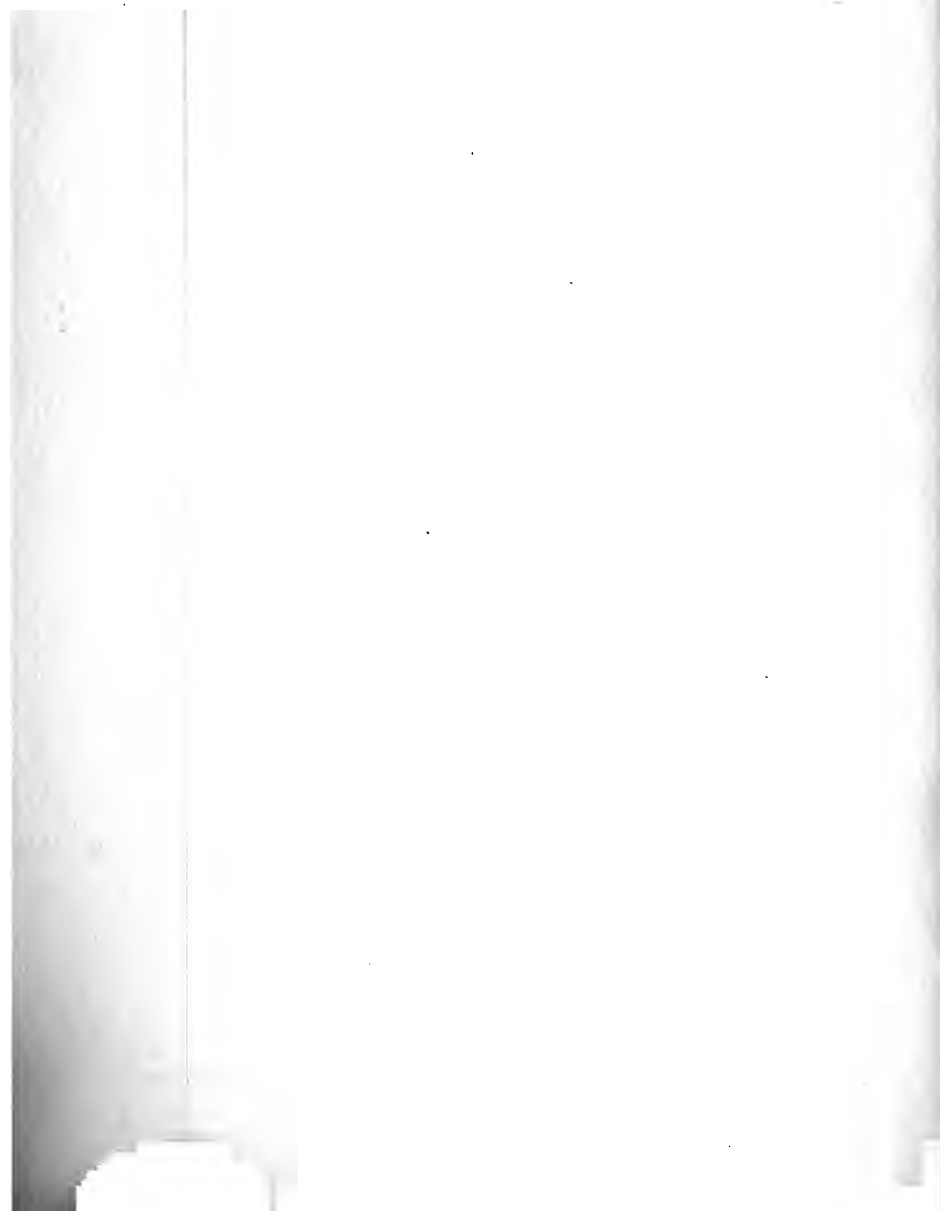


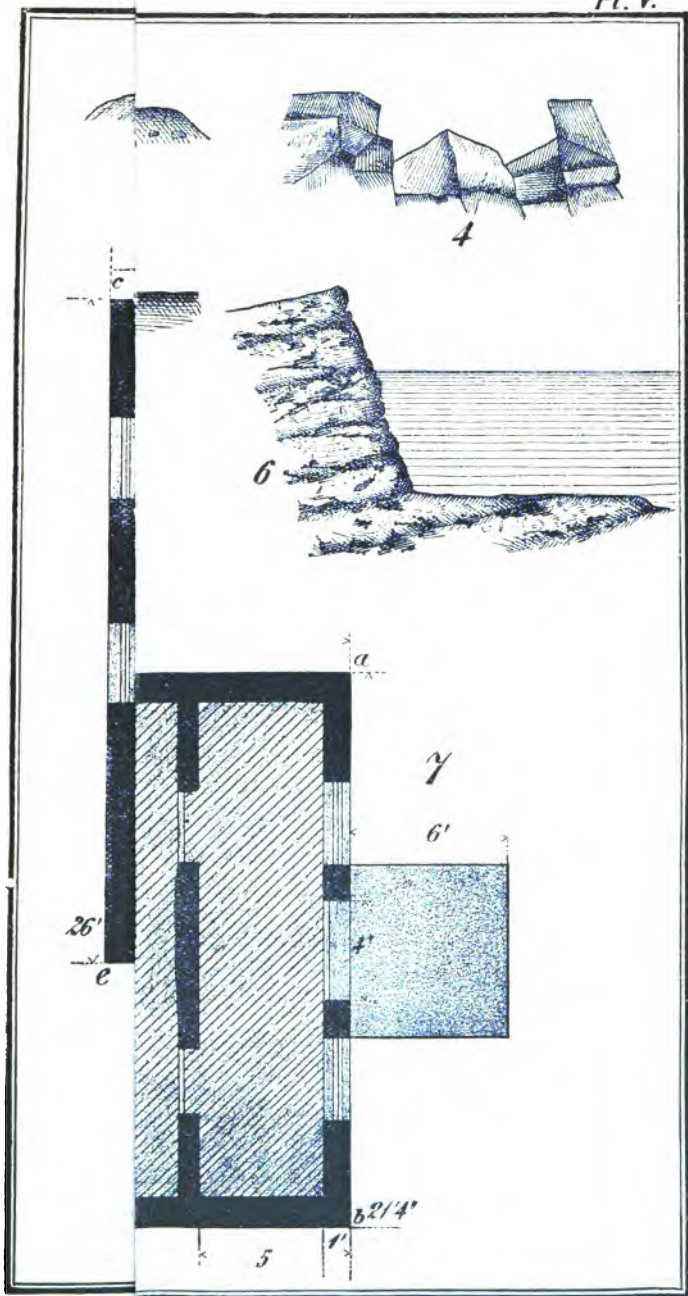
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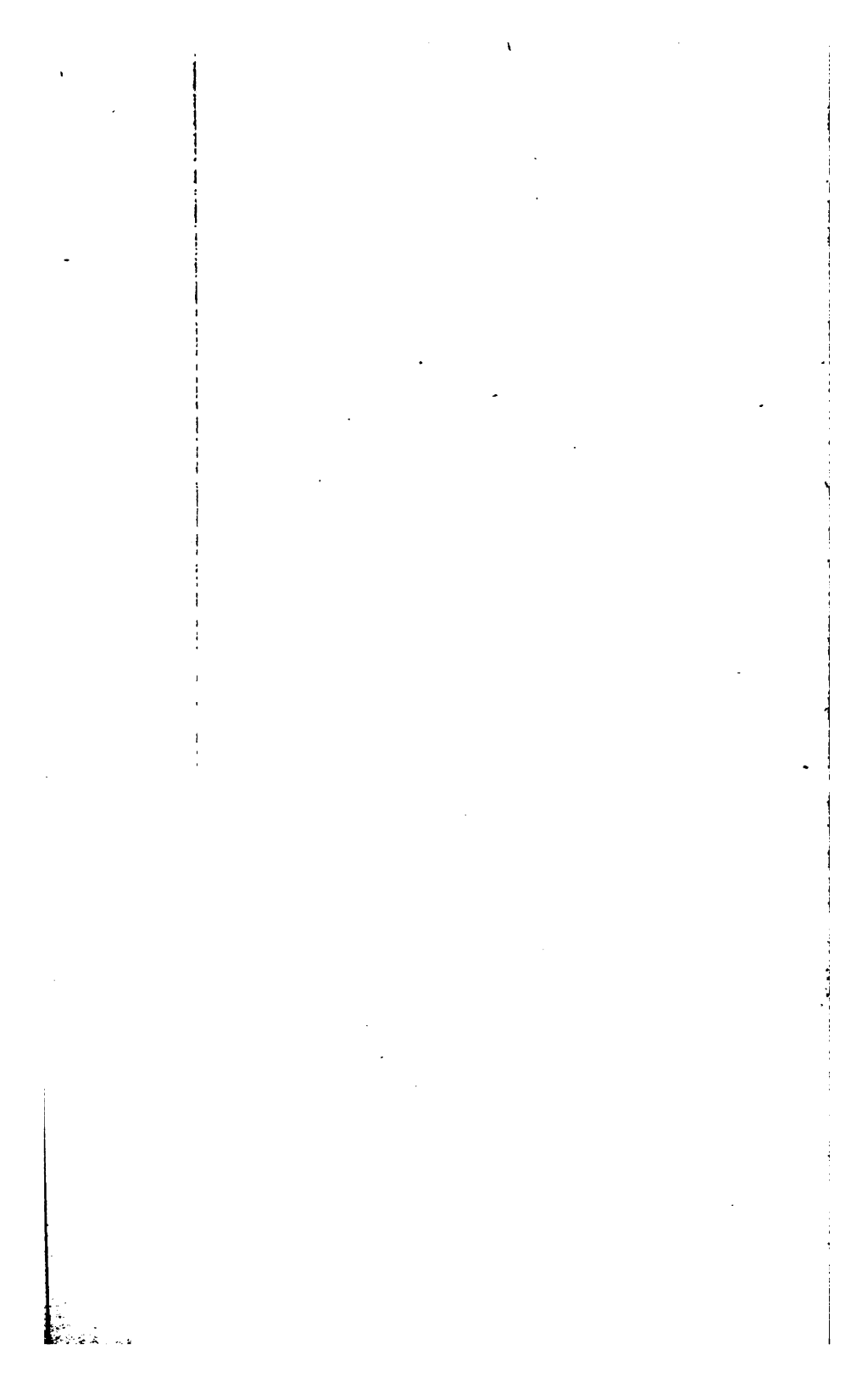


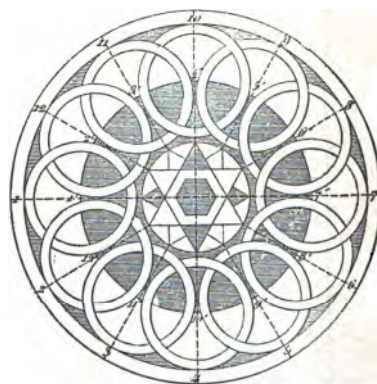
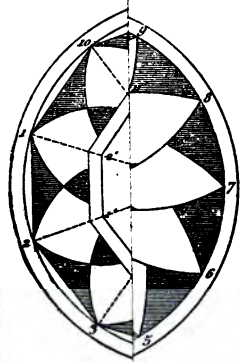
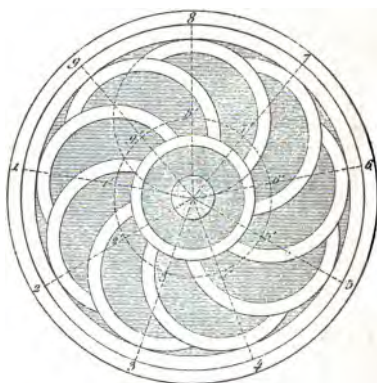
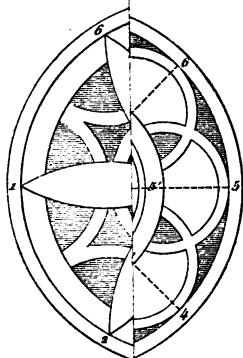
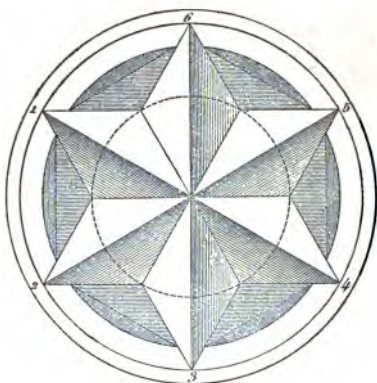
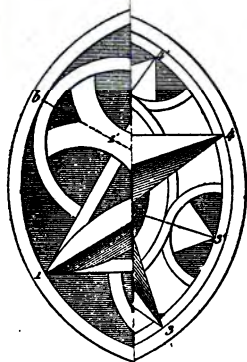


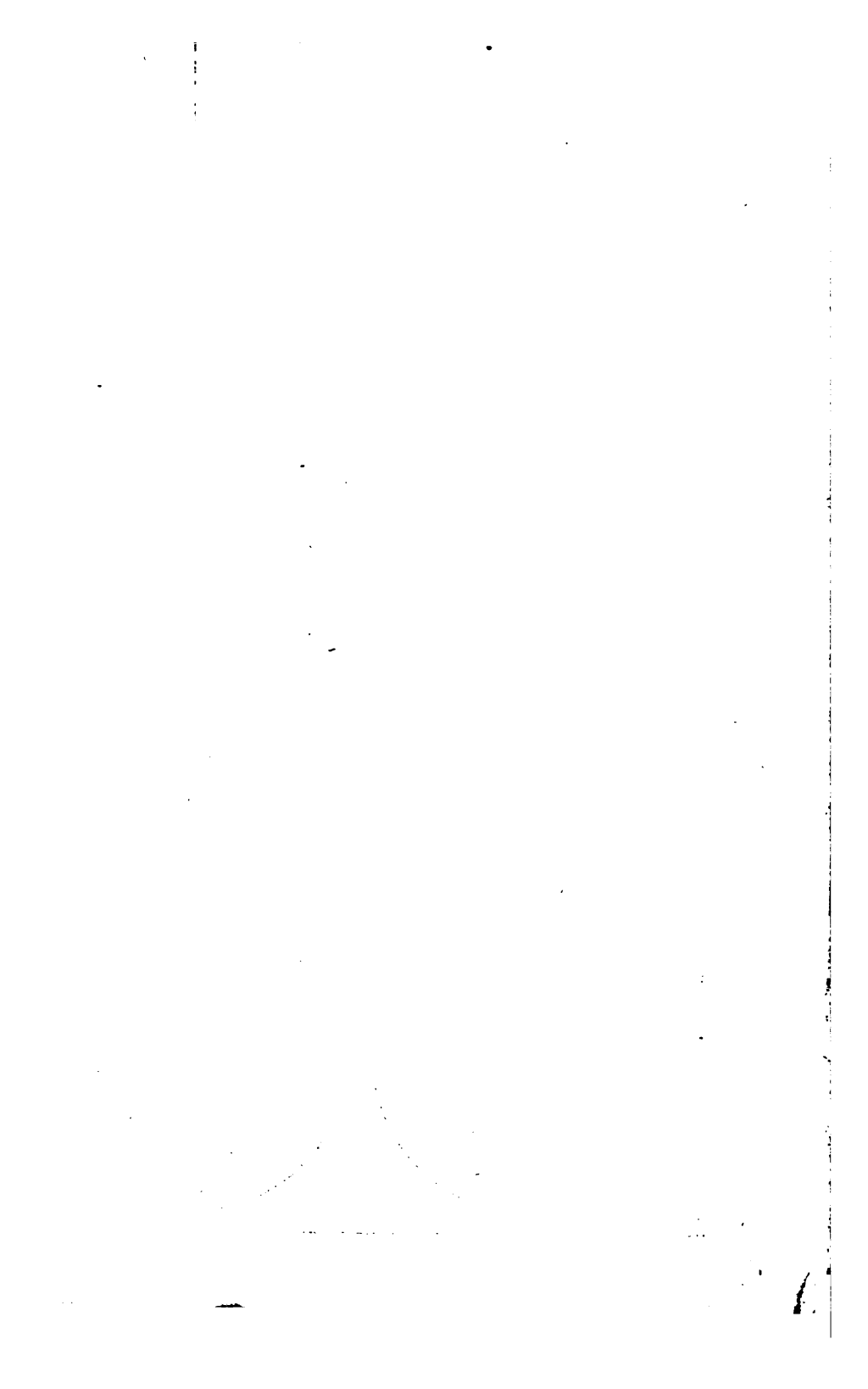


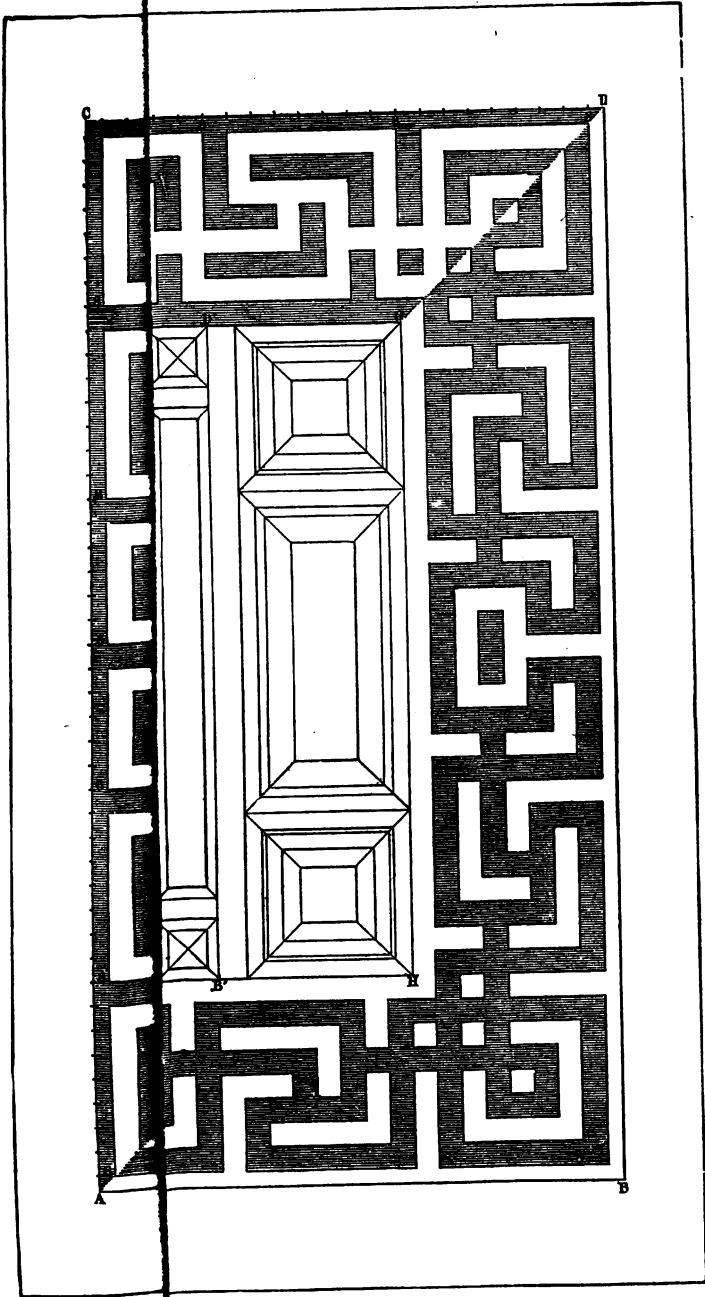


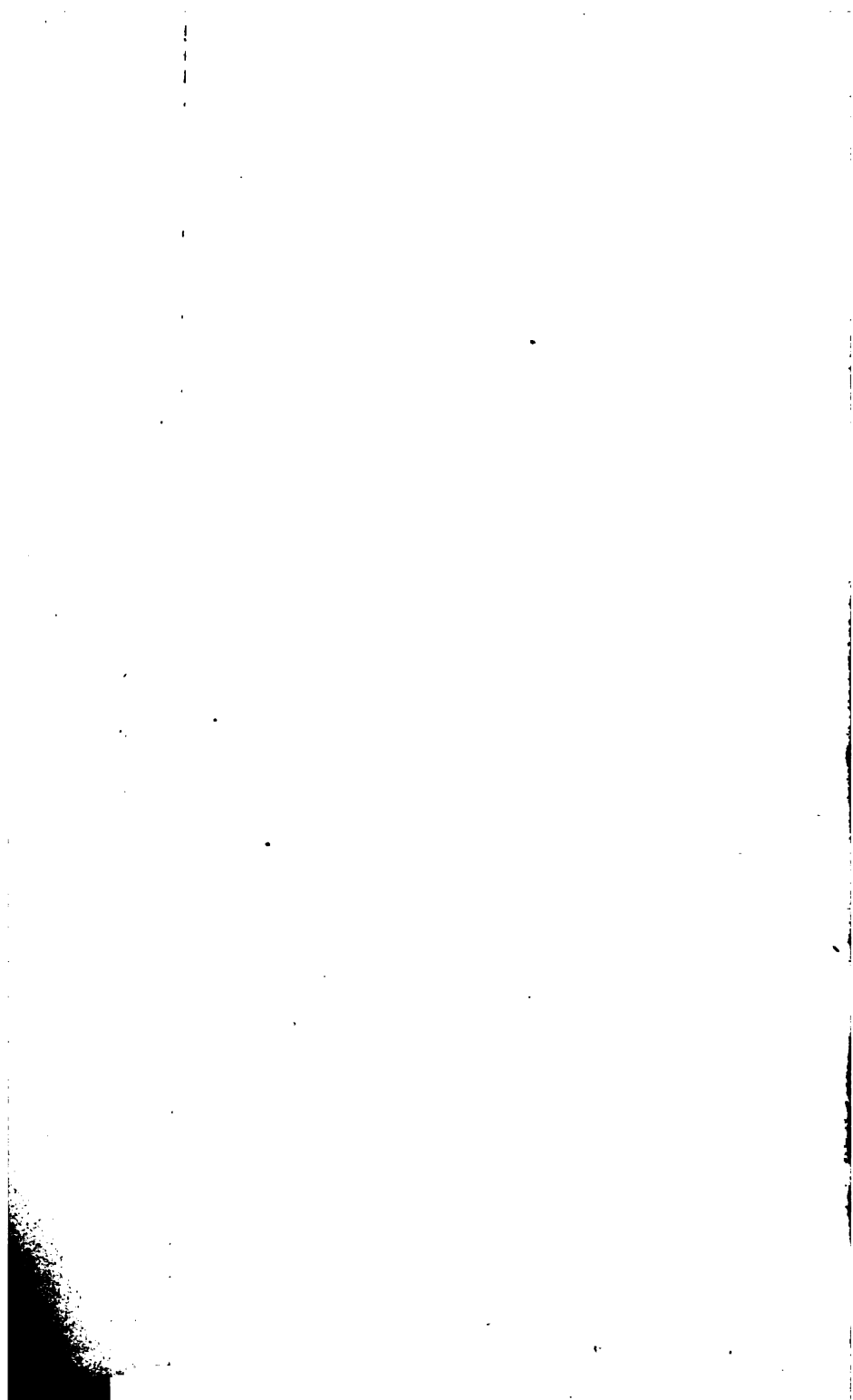


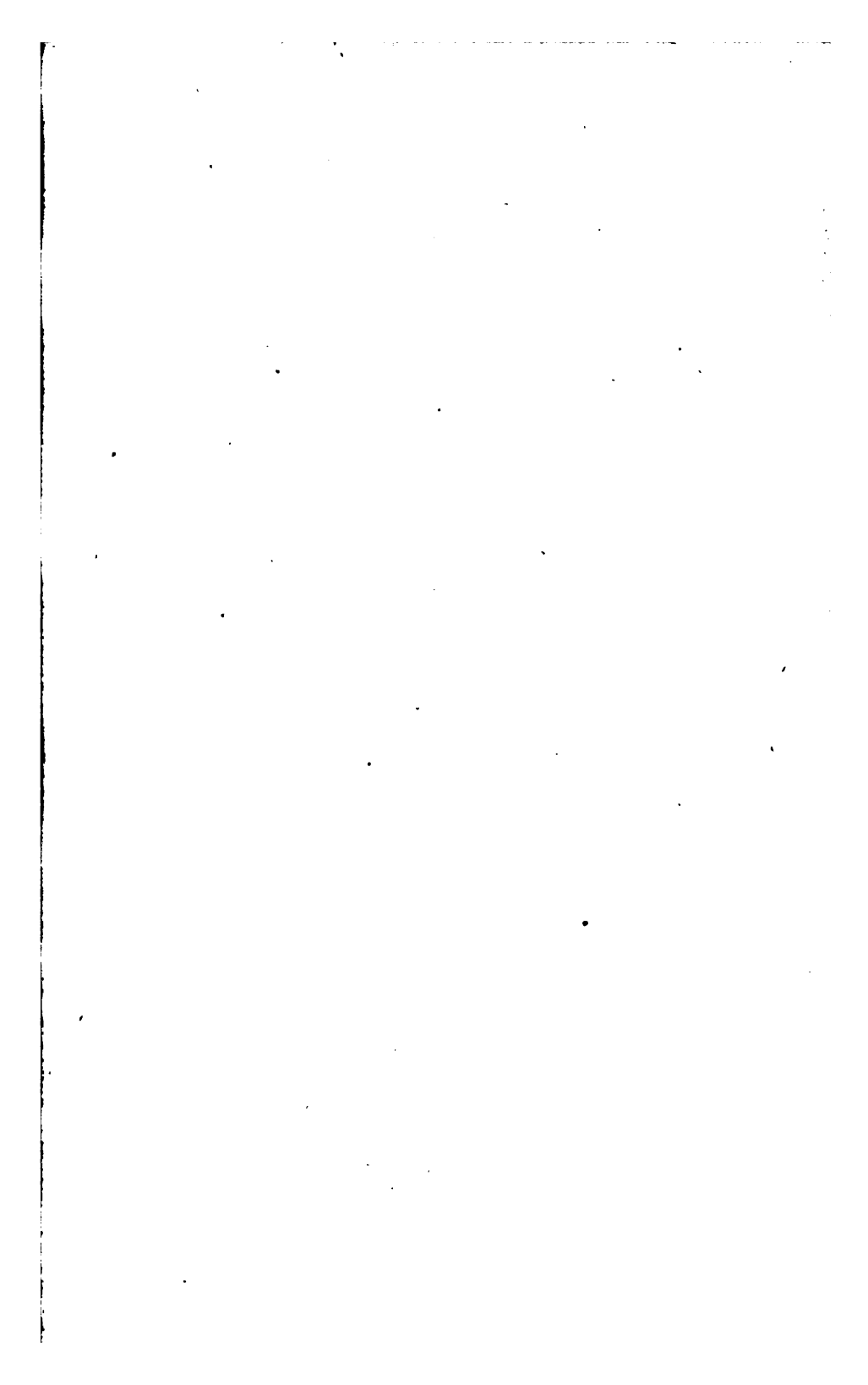
















the 1990s, the number of people in the UK who are employed in the public sector has increased by 1.5 million, from 2.5 million in 1980 to 4 million in 1995. The public sector has become a major employer in the UK, and its growth has been a major factor in the overall growth of the economy.

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